MULTIFIDELITY DESIGN OF STIFFENED COMPOSITE PANEL WITH A CRACK

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1. Abstract

In designing an aircraft composite fuselage and wing panels, one of the most important concerns is crack propagation in the skin of the stiffened panel. This study attempts to solve the weight optimization problem of a stiffened composite panel, subjected to crack propagation constraints, using two optimization strategies to satisfy the crack propagation requirements. A combination of a high fidelity method of analysis and a low fidelity method were utilized to describe the crack propagation constraint. The first strategy employs correction response surface to relate the high fidelity models to the low fidelity models. The second method converts a constraint on the stress intensity factor into an equivalent strain constraint and solves the problem through an iterative process. The equivalent strain was found to be more accurate and computationally attractive.

2. Key Words

Correction Response Surface, Crack Propagation, Equivalent Strain Constraint, Stiffened Panel

3. Introduction

The next generation of supersonic transport planes are expected to fly at very high altitudes and are very vulnerable to pressure loss due to large cracks, such as due to turbine blade penetrations. Consequently, accurate prediction of crack propagation is a major issue in the structural design of such aircraft. Exact calculation of crack propagation requires detailed structural models for computing stress intensity factors, which is computationally too expensive to be included in the structural design optimization.

The problem of optimization with high cost models is common to many engineering design problems. Recently, there has been growing interest in taking advantage of the simple models that were in use a generation ago for alleviating the computational cost problem. These models, which are less accurate, and are termed here *low fidelity models,* are also much less expensive computationally. Combining the low fidelity models with the more accurate but expensive high fidelity models can provide a good combination of high accuracy and low cost.

The general approach of such *multifidelity* techniques is to use the ratio or difference between the low fidelity and high fidelity models at one or several points in order to correct the low fidelity model at other points. For example, Haftka [1] and Chang et al.[2] suggested the calculation of the ratio and its derivatives at one point in order to construct a linear approximation of the ratio at other points in design space.

Rather than using derivative based approximation for the ratio of the low fidelity and high fidelity results, fitting the ratio as a response surface with low order polynomial has the advantage that it can apply over larger regions in the design space and also smooth out numerical noise. This *correction response surface* approach was used for approximating aerodynamic drag by Hutchison et al. [3], and by Mason et al. [4] for approximating stresses. Vitali et al. [5] used strength-of-materials models to predict stresses and buckling loads in a hat-stiffened panel, and used correction response surface technique to predict the results from the high fidelity model based on STAGS [6] FE code. Vitali et al. [7] [8] demonstrated the validity of the use of a correction response surface for calculating the stress intensity factors for cracks in stiffened panels. In the present paper, in addition to using a correction response surface, the response surface methodology is extended to also approximate the low fidelity model.

The stress intensity factors were computed using both high and low fidelity models. The high fidelity model describes the crack in the stiffened panel with great detail and computes the stress intensity factor using classical linear elastic fracture mechanics. The low fidelity model uses a coarse finite element model, and the crack is not present in the model. The stress intensity factor is calculated from a closed-form solution for stress intensity factors.

In one approach, a quadratic correction response surface is fitted to the values of the ratio of the high fidelity and low fidelity stress intensity factors. Further, a cubic response surface is constructed for the corrected low fidelity stress intensity factors in terms of the design variables. The cubic response surface is used as constraint in the design optimization problem.

A second approach to the same problem is the use of an equivalent strain constraint. The equivalent strain constraint is inspired by a similar approach used in an in-house Ford Motor Company program STOPFAT (Structural Thickness Optimization Procedure based on FATigue life). This approach converts the fatigue life constraint, which is not available in general finite element programs, such as NASTRAN, to an equivalent stress constraint, which is commonly available in such programs. Here we do the same with the stress intensity constraint, converting it to an equivalent strain constraint that can be inexpensively optimized with a low fidelity model.

2. Problem Description

The dimensions of the composite stiffened panel to be designed are shown in Fig. 1. The composite stiffened panel was subjected to a tensile load case of 2000 lb/in. The material used in the panel was AS4/3501-6 graphite/epoxy, and its properties are given in Table 1. The geometry of the composite stiffened panel and the size of the crack ($2a$ = 4.0 inches) were kept constant throughout the study. The stacking sequence of the skin and the stacking sequence of the stiffener were kept the same. The lay-up used for both the skin and the stiffener of the panel was $[\pm 45/90/0]_s$.

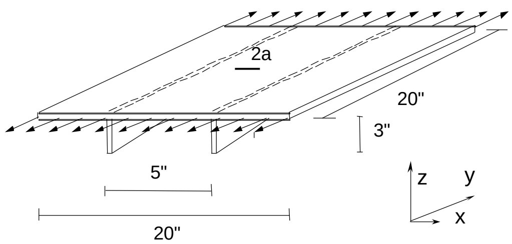

Figure 1: Panel geometry

Material property	Value
Young's modulus E_1	20 10^6 psi
Young's modulus E_2	1.4 10^6 psi
Shear modulus G_{12}	0.76 10^6 psi
Poisson's Ratio ν_{12}	0.30
Density ρ	0.063 lb/in^3
Kq^0 (Fracture toughness)	100.000 psi $\sqrt{in}$

Table 1: Material properties of AS4/3501 -6

. The thicknesses of different plies, $\pm$45, 90 and 0, present in the skin and in the stiffener were used as design variables. The thicknesses of the +45 and -45 plies in each laminate were set to be equal to each other. A total of six ply thickness design variables were used and were labeled as: *p45*, *p90*, *p00*, *b45*, *b90*, *b00*. The last two digits indicate the ply angle while the letter "p" and the letter "b", respectively, refer to the plate (skin) or the blade (stiffener). All design variable values ranged between 0.005 inches and 0.025 inches. The lower limit reflects that the material system used in this study, the AS/3501-6, is generally available in the form of prepreg tape with a nominal thickness of 0.005 inch. The maximum values represent estimates of the required material based on the papers by Vaidya et al. [9], [10].

Finite Element Models

The stiffened panel is simply supported along the edges parallel to the x–axis, and free along the edges parallel to the y–axis (see Fig. 1). A multipoint constraint was applied to the skin and to the stiffener, enforcing the same displacement in the y-direction along the simply supported edges. Two different finite element models of the stiffened panel were created using the STAGS program and the GENESIS [11] program. In both finite element models, symmetry was utilized in order to model only ¼ of the structure, thereby reducing the total number of degrees of freedom.

The first STAGS computer model of the stiffened panel was used to accurately capture the stress gradients that develop in the region near the crack tip, when the structure was subjected to the tensile loading. In order to achieve accuracy, a large number of 4-noded elements were employed in the vicinity of the crack tip. The refined model consisted of 7,158 elements and 46,680 degrees of freedom and is shown in Fig. 2.

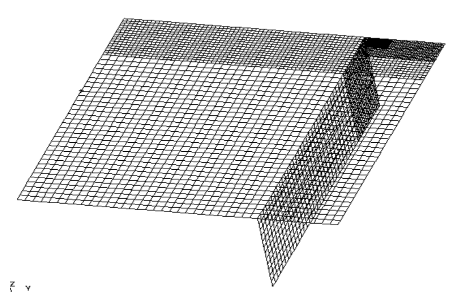

Figure 2: Refined element model ¼ panel

The second finite element model was coarser since it did not contain a crack. A uniform mesh of 4-noded elements was employed. The model was used to compute the nominal strains and stresses in the location where a crack is assumed to be. Then, a closed form solution was used to compute the approximate stress intensity factor in the presence of the crack. The low fidelity model used 2080 elements for a total of 13284 degrees of freedom.

4. Stress Intensity Factor Calculation

The method of Vaidya and Sun was used for predicting crack propagation in a composite laminate. In this method the average stress intensity factor, *K*, through the thickness of the plate is computed by modeling the laminate as an equivalent orthotropic plate. Then the stress intensity factor in the zero-degree ply is computed using

$$K^0 = ?\ K \quad (1)$$

where the factor *h* is defined as the ratio of the nominal stress in the zero-degree ply to the average stress in laminate:

$$? = \frac{s^0_{yy}}{s_{yy}} \quad (2)$$

The crack in the skin of the panel is assumed to propagate, leading to structural failure, when stress intensity factor in the 0^0 plies K^0 reaches K^0_q, the fracture toughness of the 0^0 ply. The value of the fracture toughness K^0_q was taken from Vaidya and Sun[5] the theoretical distribution of s_{yy} near the crack tip of the equivalent orthotropic panel is given by:

$$s_{yy} = \frac{K}{\sqrt{2\,p\,r}} \quad (3)$$

The high fidelity value of *K*, K_{HF}, was obtained by fitting s_{yy} as a function of the distance from the crack tip, *r* using Eq. (3) in a least square sense. Calculation of one K_{HF} required 556 seconds of usage time and 41.9 seconds of CPU time on a 233 MHz DEC Alpha Station 200 4/233.

The low fidelity stress intensity factor K_{LF} was also calculated using a low fidelity method, based on borrowing the infinite cracked plate solution for *K*

$$K = s_{yy}\sqrt{p\,a} \quad (4)$$

where s_{yy} in (4) is the average stress present in an uncracked equivalent orthotropic panel at the location of the crack is assumed to be present. Calculation of one value of K_{LF} required 66 seconds of usage time and 7.8 seconds of CPU time on a 233 MHz DEC Alpha Station 200 4/233. The low fidelity value of *K* obtained is approximate mainly for two reasons: (i) the finite size of the panel was not taken into consideration in Eq. (4); and (ii) the beneficial effect of the stiffener on reducing the stress intensity factor could not be fully captured by this model. However, the low fidelity method had the advantage of being computationally inexpensive.

5. Optimization via Correction Response Surface

Both high fidelity and low fidelity models were employed in construction of the response surface that described the crack propagation constraint. This was done in an attempt to combine the accuracy of the high fidelity method with the low computational cost of the low fidelity method.

First an orthogonal array using three levels for the six design variables, was used to select a total of 54 design points, with the “addelkemp3” subroutine, written by Owen [12] and based by Addelman et al [13]. For these 54 design points, the value of the stress intensity factor *K* of the equivalent orthotropic structure was calculated using both the high fidelity and low fidelity methods. The ratio,

$$ß = \frac{K_{HF}}{K_{LF}} \quad (5)$$

was then computed at all 54 design points. A quadratic response surface b^{Quad} for the ratio *b* in the six design variables was fitted through the 54 design points. The ratio varied between 0.81 and 1.07 indicating moderate differences between the two models. The largest differences were for design points that had a very thin skin and a very thick stiffener.

A full quadratic response surface in six design variables normally has 28 terms; however, some of the quadratic terms were eliminated in order to improve the prediction accuracy of the response surface. In order to choose the terms to be eliminated the mixed mode (both backward and forward elimination) stepwise regression method of JMP [14] program was used. Using this procedure, a total of 19 terms were retained by the response surface. The fit was quite good as seen from Table 2.

Next, a 5 level orthogonal array in the six design variables was created for 250 design points. Using the low fidelity analysis, the value of K_{LF} for the selected 250 design points was found and corrected by multiplication by b^{Quad}. A cubic response surface K^{cubic} in the six design variables was fitted to the values of the corrected values of K_{LF} with the mixed mode stepwise regression selecting 64 of the possible 84 original terms. The statistical parameters of the fitting procedure are presented in Table 3.

Parameter	Value
Mean of Response	0.976
Root Mean Square Error	0.0034
RSquared Adjusted	0.987

Table 2: b^{Quad} **response surface fit parameters**

Parameter	Value
Mean of Response	33439
Root Mean Square Error	447
RSquared Adjusted	0.998

Table 3: Corrected Low Fidelity K^{Cubic} response surface fit parameters

The accuracy of the cubic response surface was assessed by calculating the value of the stress intensity factor *K* at some additional design points not used in the construction of the response surface. It was found that the response surface had a maximum error of about 16%. The value of the stress intensity factor in the 0^0 plies could then be estimated by using Eq. (1)

$$K^0 = ? \quad K^{Cubic} \tag{6}$$

The goal of the optimization problem was to find the minimum weight design of a stiffened composite panel, as described by six design variables, that satisfy the crack propagation constraint of $K^0 \leq K_q^0$.
The optimization problem thus obtained is summarized in Table 4.

Objective Function:	Min{Weight}
Design Variables :	p45, p90, p00 b45, b90, b00
Constraints:	0.005 < p45 < 0.025
	0.005 < p90 < 0.025
	0.005 < p00 < 0.025
	0.005 < b45 < 0.025
	0.005 < b90 < 0.025
	0.005 < b00 < 0.025
	$K^0 < 100{,}000$ psi $\sqrt{in}$

Table 4: Optimization problem

Parameter	Value
p45	0.0050 inches
p90	0.0050 inches
p00	0.0119 inches
b45	0.005 inches
b90	0.005 inches
b00	0.025 inches
K^0	99936 psi $\sqrt{in}$
Weight	0.443 lb

Table 5: Optimum solution from correction response surface approach

The optimization problem was implemented and solved using Microsoft EXCEL. An optimum solution was found and the corresponding values are presented in Table 5. The value of stress intensity factor K for the optimum design structure was checked using the high fidelity analysis. The K value of the equivalent orthotropic panel obtained from the high fidelity finite element model was $K_{HF} = 55001$ psi$\sqrt{in}$. The value of ***h*** for the optimum design was equal to ***h*** = 1.835. The stress intensity factor in the 0^0 plies K^0, could then be calculated as:

$$K^0 = ? \; K_{HF} = 101{,}264 \text{ psi}\sqrt{in} \tag{7}$$

The response surface predicted the final value of the stress intensity factor in the zero plies with an accuracy of 1.3%. The design strategy of the response surface was to increase the value of *b00* as much as possible so to allocate the largest portion of the load to the stiffener, and only then it increased the value of *p00* in order to increase the crack strength of the plate. For this level of load all the other four design variables remained at its minimum sizes.

6. Optimization Problem via Equivalent Strain Constraint

For composite laminates, experimental results obtained by Vaidya and Sun show that while matrix cracks provide stress relaxation, the final fracture of the laminate is controlled by fiber breakage in the *0°* plies. For a given load and a given laminate configuration the value of the allowable stress resultant $N_{y,All}$ far away from the crack can be calculated as

$$N_{y,All} = \frac{K_q^0}{K^0} N_y \tag{7}$$

where K^0 is the stress intensity factor generated by the remotely applied stress resultant N_y that can be calculated by the high fidelity method and K_q^0 is the fracture toughness of the material. Applying the allowable $N_{y,All}$ to a panel that does not model the crack the allowable strains in the *0°*, *45°* and *90°* fiber principal directions in the location where the crack is assumed to be can be calculated as

$$\left\{e_{All}^{q,\mathrm{Prin}}\right\} = \left[R(q)\right]\left[A\right]^{-1}\left\{N_{All}\right\} \tag{8}$$

where the left hand side of Eq. (8) represents the strains in the principal directions of the laminae at an angle q, $R(q)$ is the rotation matrix from the global to the laminae coordinates, A^{-1} is the inverse of the laminate stiffness matrix, and N_{All} is the allowable stress resultant applied. The obtained strain values are then used as strain allowables that are equivalent to the stress intensity factor constraint.

For a given stiffened panel we calculate K_{HF} and the corresponding strain allowables. Next the structure is optimized with the strain constraint, a new design is obtained and the procedure is repeated. The equivalent strain constraint will change from one iteration to the next for two reasons. First, the ratio of zero degree plies varies from one iteration to the next varying the parameter h of Eq. (2) that is needed to calculate K^0. Second, the structure used in the equivalent strain method does not model the crack and therefore cannot capture the effect that the stiffener has in the region near the crack tip. Since the STAGS program does not have optimization capability, the optimization was carried out with the GENESIS program.

Parameter	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5
p45 inches	0.0250	0.0050	0.0050	0.0050	0.0050
p90 inches	0.0250	0.0050	0.0050	0.0050	0.0050
p00 inches	0.0250	0.0127	0.0178	0.0132	0.0118
b45 inches	0.0250	0.0050	0.0050	0.0050	0.0050
b90 inches	0.0250	0.0050	0.0050	0.0050	0.0050
b00 inches	0.0250	0.0250	0.0051	0.0250	0.0245
η	2.57	1.79	1.59	1.76	1.84
K^0 psi $\sqrt{in}$	46860	95532	105580	93456	99912
p45, b45 ε^1, ε^2	$7.33\ 10^{-5}$	$7.62\ 10^{-5}$	$7.17\ 10^{-5}$	$7.64\ 10^{-5}$	-
p45, b45 ε^{12}	$2.70\ 10^{-4}$	$2.87\ 10^{-4}$	$2.70\ 10^{-4}$	$2.88\ 10^{-4}$	-
p90, b90 ε^1 p00,b00 ε^2	$6.50\ 10^{-5}$	$6.74\ 10^{-5}$	$6.33\ 10^{-5}$	$6.76\ 10^{-5}$	-
p90, b90 ε^2 p00, b00 ε^1	$2.11\ 10^{-4}$	$2.20\ 10^{-4}$	$2.06\ 10^{-4}$	$2.21\ 10^{-4}$	-
Weight lb	1.482	0.452	0.442	0.459	0.441

Table 6: Equivalent strain iterations constraint optimum

The iteration history of the optimization problem presented in Table 4 using the strain equivalent method is presented in Table 6. As can be seen from Table 6, the solution obtained from the equivalent method is slightly more accurate than the solution obtained using the correction response surface. Furthermore the method converged in only five iterations, requiring only five high fidelity analyses. Moreover, the equivalent strain method showed the existence of an alternative design very close in weight to the optimum one in the third iteration. The weight of the structure of iteration 3 and iteration 5 are very close but the design philosophy is different. In the Iteration 3 design the optimization procedure reduced the size of the stiffener in order to increase the crack propagation strength of the panel. The optimum weight design obtained at Iteration 5 increases the thickness *b00* in order to carry a larger portion of the load but at the same time it slightly increases *p00* in order to increase the strength of the panel. The optima found by the two methods do not differ very much in weight (note that the 6% constraint violation will require some increase in weight) but they differ in the design philosophy and in the computational cost

8. Concluding Remarks

In many engineering design problems optimization requires computationally expensive analyses. In order to overcome this difficulty more attention has been recently given to simple models that were used as design tools a generation ago. These models are often referred as low fidelity models and if not extremely accurate in nature they are computationally very attractive. Combining the low fidelity models with more accurate but expensive high fidelity models it is possible to achieve of high accuracy at a low cost.

In this paper an example of a composite stiffened panel with crack propagation constraints is used to demonstrate two different methodologies that use a low fidelity model and couple it with a high fidelity model. The first approach used a correction response surface in order to correct the low fidelity values of the stress intensity factor for an orthogonal set of design points. The correction response surface was calculated fitting a quadratic polynomial to the ratio of the stress intensity factors obtained from a high fidelity model and from a low fidelity model. Once the low fidelity values were corrected, a cubic response surface in the six design variables was fitted through the entire design space. This cubic response surface was used as a constraint in the optimization process.

The second approach used the concept of equivalent strain constraint to design the stiffened panel against crack propagation. The constraint on the stress intensity factor was converted into an equivalent strain constraint applied to the low fidelity model. The solution was found in an iterative fashion where the equivalent strain constraint is updated after each optimization by using the high fidelity model. A minimum weight design was found in five iterations. In addition the equivalent strain constraint method did not require any derivatives of the stress intensity factor with respect to the design variables.

9. Acknowledgments

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