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# Engineering Notes

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## Minimum Mass Design of Insulation Made of Functionally Graded Material

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### I. Introduction

METAL foams[1] are candidates for advanced thermal protection systems[2] (TPS) for future reusable launch vehicles. Such multifunctional structures would insulate the vehicle interior from aerodynamic heating as well as carry primary vehicle loads. Varying the density, geometry, and/or material composition from point to point within the foam can produce functionally graded materials (FGM) that might be superior to uniform materials.

Bhattacharya et al.[3] investigated the heat transfer in metal foams based on a model consisting of two-dimensional array of hexagonal cells. Boomsma and Poulikakos[4] developed a model for heat conduction in fluid-saturated metal foams with tetrakaidecahedron cell geometry. In this Note we use a simple heat-transfer model developed by Glicksman[5] for foams. The model is used to derive the optimality condition and demonstrate that the insulation mass can be minimized by using graded foams. Venkataraman et al.[6] developed a criterion for minimizing heat conduction through an open-cell titanium foam with variable cell size through its thickness. The current study seeks to identify density profiles that can yield improvements in weight efficiency compared to materials with uniform density. These results can then be used to direct research into improved modeling of FGM that will be used to refine the initial optimization. The results could also be used to select promising TPS configurations for testing. The TPS problem is inherently a transient one. However, we solve the simpler steady-state problem to gain understanding of the effects of using functionally graded insulations.

### II. Effective Thermal Conductivity of Metal Foam

For the purpose of deriving the homogeneous properties, the foam is idealized as having rectangular cells of uniform size. We then obtain the effective conductivity $k$ as a function of local volume fraction. Heat transfer through porous metal foams involves a number of heat-transfer modes. Equations used in this Note for gas conduction, metal conduction, and radiation are developed by Venkataraman et al.[6] Convection effects, which are much smaller than the aforementioned modes, are neglected. For a perfect cubic cell, the volume fraction $V_f$ of metal in the open-cell foam, referred to as solidity, is given by

$$V_f = \tfrac{3}{4}\pi(d_s/a)^2 \tag{1}$$

where $a$ is the size of the unit cell and $d_s$ is the strut diameter.

The overall effective thermal conductivity as function of temperature and volume fraction consists of the contributions of all modes of heat transfer. We assume that the medium is optically thick. The optical thickness is defined as the ratio of the characteristic length to photon mean free path, and for the metal foams used in our study it is always larger than 10. Consequently, the contributions of the three modes of heat transfer, that is, gas conduction, metal conduction, and radiation, can be linearly combined as

$$k(T, V_f) = (1 - V_f)k_g + k_m + k_r \tag{2}$$

Expressions for $k_g$, $k_m$, and $k_r$ can be found in Venkataraman et al.[6]

The variation in density is achieved by tailoring the cell size while keeping the strut diameter fixed at 0.05 mm (0.002 in.). For dense foams, cell size is small; heat transfer is dominated by conduction, and so the effective conductivity is a monotonic function of solidity. For higher temperatures or low-density foams, radiation is more important, and larger cell size increases the radiative heat transfer so that the effective thermal conductivity increases with decrease in foam density. This leads to a minimum value of effective thermal conductivity as the density is varied from the minimum to the maximum allowable value.

### III. Optimality Criterion for Minimum Mass

We consider an insulation panel of thickness $h$ with a given heat flux $Q_0$ and temperature limits $T_m$ on its hot side ($x = h$) and a given temperature $T_0$ on its cool side ($x = 0$), as shown in Fig. 1. For the purpose of illustration, the temperatures $T_0$ and $T_m$ are assumed as 400 and 1500 K, respectively. These temperatures, respectively, are the maximum temperatures the structure and the TPS can withstand. We assume that there is very little heat transfer in the plane of panel, so that the problem can be treated as one dimensional. As mentioned earlier, we consider the steady-state heat conduction in this Note.

The steady-state heat transfer equation and boundary conditions are

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = 0 \tag{3a}$$

$$k\frac{\partial T}{\partial x}\bigg|_h = Q_0 - \sigma T^4(h) \tag{3b}$$

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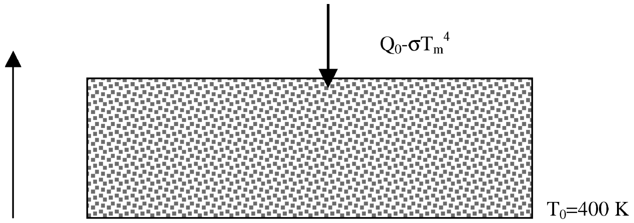


**Fig. 1 Schematic of heat transfer in insulation. $Q_0$ is the applied aerodynamic heating, and $\sigma T_m^4$ represents the heat radiated back into space.**

$$T(0) = T_0 \tag{3c}$$

where $\sigma$ is the Stephan–Boltzmann constant. Later we will use the constraint

$$T(h) \leq T_m \tag{3d}$$

in order to determine the thickness $h$ of the insulation. Actually we will use the equality constraint $T(h) = T_m$ for it will reduce the amount of heat transferred to the insulation by increasing the radiated heat. Combining Eqs. (3a), (3b), and (3d), we obtain the governing equation as

$$k\frac{\partial T}{\partial x} - Q_0 + \sigma T_m^4 = 0 \tag{3e}$$

For the purpose of the illustration of our procedure, we have assumed the emittance of the surface as unity. In reality there will be a facesheet covering foam. If we assume that the facesheet is very thin and has very high conductivity, then its thermal resistance can be neglected.

The density of the foam $\rho$ is given by $\rho = \rho_m^* V_f$, where $\rho_m^*$ is the mass density of the strut material and $V_f$ is the solidity of the foam. The mass per unit area of the TPS $m$ is

$$m = \rho_m^* \int_0^h V_f(x)\,\mathrm{d}x \tag{4}$$

Our objective is to minimize $m$ subject to the constraint given by Eq. (3e). Using a Lagrange multiplier $\lambda(x)$ for the constraint, the necessary conditions for the optimum are obtained by looking for stationary points of the Lagrangian $L$

$$L = \int_0^h F(x, T, V_f, T')\,\mathrm{d}x \tag{5}$$

where

$$F = V_f + \lambda(x)\big(kT' - Q_0 + \sigma T_m^4\big) \tag{6}$$

and $(\cdot)' = \mathrm{d}(\cdot)/\mathrm{d}x$.

The Euler–Lagrange equations corresponding to Eq. (5) are

$$\frac{\partial F}{\partial V_f} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial F}{\partial V_f'}\right) = 0, \qquad \frac{\partial F}{\partial T} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial F}{\partial T'}\right) = 0 \tag{7}$$

Substituting for $F$ in Eq. (7), we obtain

$$1 + \lambda T'\frac{\partial k}{\partial V_f} = 0, \qquad \lambda T'\frac{\partial k}{\partial T} - (\lambda k)' = 0 \tag{8}$$

Eliminating $\lambda(x)$ from Eq. (8) and also using the governing equation [Eq. (3a)], we obtain the optimality condition as

$$V_f' = -\left(\frac{k}{\partial k/\partial V_f}\right)' \tag{9}$$

Integrating the preceding equation, we derive the optimum solidity profile as

$$V_f(x) = V_{\mathrm{fc}} - \frac{k}{\partial k/\partial V_f} \tag{10}$$

where $V_{\mathrm{fc}}$ is a constant to be determined by the condition $T(h) = T_m$ [see Eq. (3d)]. For a given temperature $T$ and the integration constant $V_{\mathrm{fc}}$, Eq. (10) and $k(T, V_f)$, which is known, determine the optimum solidity $V_f^*$ and the optimal $k^*$ independently of the position $x$.

## IV. Numerical Evaluation of Designs That Satisfy the Optimality Criterion

Equation (10) is solved for the optimal $k^*$ and $V_f^*$ for a range of values of $V_{\mathrm{fc}}$ and $T$, and the results are shown in Fig. 2. The range of $V_{\mathrm{fc}}$ was chosen such that the values of insulation thickness obtained would be in the useful range for the example. Figure 2 indicates that there might be two values of $V_f$ for a given $k$, but for minimum mass we obviously select the lower value, that is, values of $V_f$ where $\mathrm{d}k/\mathrm{d}V_f \leq 0$.

The values of the optimal conductivity $k^*$ shown in Fig. 2 were fitted as quadratic polynomials of the temperature for a range of values of $V_{\mathrm{fc}}$ (hence thickness) as

$$k^* = k^*(T) = a_1T^2 + a_2T + a_3 \tag{11}$$

This quadratic approximation allows analytical solution of the heat conduction Eq. (3e). Substituting the optimum $k^* = k^*(T)$ from Eq. (11) into Eq. (3e),

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{Q_0 - \sigma T_m^4}{a_1T^2 + a_2T + a_3} \tag{12}$$

Equation (12) can be integrated to obtain

$$(a_1/3)T^3 + (a_2/2)T^2 + a_3T - \big(Q_0 - \sigma T_m^4\big)x + c_0 = 0 \tag{13}$$

where

$$c_0 = -(a_1/3)T_0^3 - (a_2/2)T_0^2 - a_3T_0 \tag{14}$$

Solving Eq. (13) provides the distribution of temperature across the thickness of the panel. Repeating the procedure for different $V_{\mathrm{fc}}s$, we find the specific $V_{\mathrm{fc}}$ that satisfies $T(h) = T_m$. With the optimal temperature profile $T^*(x)$ and optimal effective conductive profile $k^*$, we can find the optimal solidity profile $V_f^*(x)$ by solving $k^* = k(T^*, V_f^*)$. After we obtain $V_f^* = V_f^*(x)$, we substitute it into Eq. (4) to find the mass per unit area of insulation. Figure 3 shows the optimal solidity distribution in a FGM for different thicknesses of insulation for the case $T(h) = T_m = 1500$ K and $Q_0 = 300{,}000$ W/m$^2$. The corresponding values of $V_{\mathrm{fc}}$ are shown in Table 1.

For the purpose of comparison, the problem of an optimum (minimum mass) TPS having a uniform solidity that satisfies Eq. (3) was also solved. The procedures are described in Venkataraman et al.[6] The areal density (mass per unit area) of the insulation is obtained as the product of thickness and solidity. Figure 4 compares the relative areal density for uniform and optimal FGM insulation for various values of thickness of the insulation. It can be seen that the mass

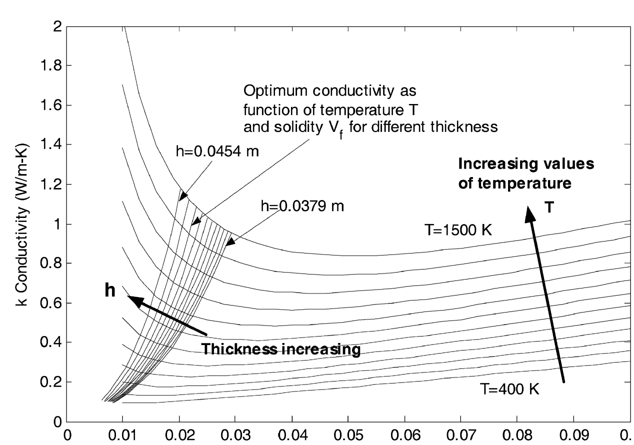


**Fig. 2 Loci of optimum solidity values for various values of thickness.**

**Table 1 Integration constants $V_{fc}$ corresponding to different insulation thicknesses**

| $V_{fc}$ ($\times 10^{-2}$) | $h$ ($\times 10^{-2}$ m) |
|---|---|
| −3.5 | 3.79 |
| −3.25 | 3.82 |
| −3.0 | 3.85 |
| −2.75 | 3.89 |
| −2.5 | 3.93 |
| −2.25 | 3.99 |
| −2.0 | 4.05 |
| −1.75 | 4.13 |
| −1.50 | 4.23 |
| −1.25 | 4.36 |
| −1.00 | 4.54 |

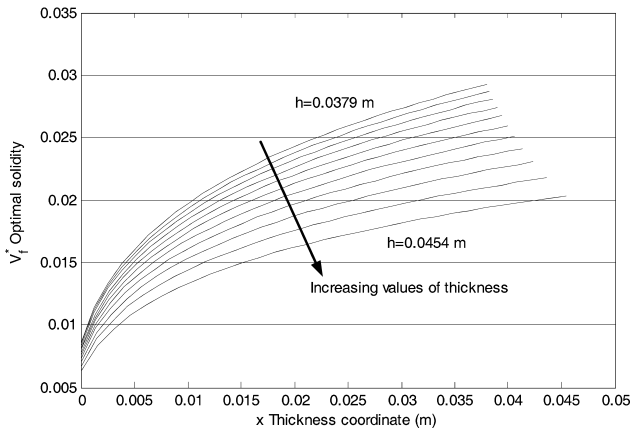


**Fig. 3 Optimum solidity profiles for various thickness of insulation.**

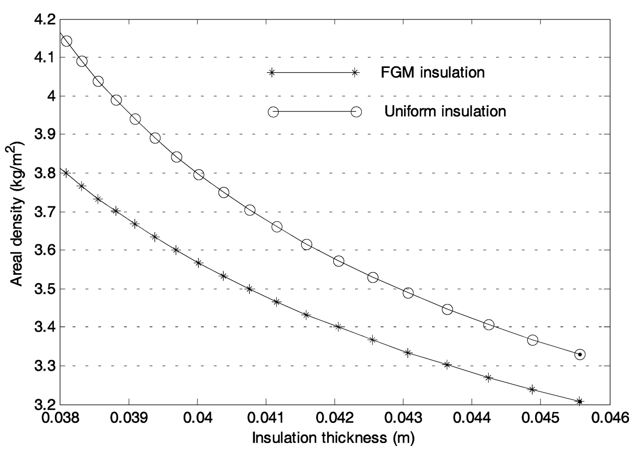


**Fig. 4 Comparison of mass per unit area for functionally graded and uniform solidity insulations.**

savings are higher for thin insulation with about 8.6% less mass for 3.79 cm and only about 3.6% savings for 4.54 cm.

## V. Summary

We studied the problem of one-dimensional steady-state heat conduction in metallic foams used as thermal protection systems with varying density in the thickness direction. The thermal conductivity of the foam is a function of temperature as well as the density, and it has a minimum value in the range of densities of our interest. An optimality criterion in the form of a differential equation was derived in order to minimize the total mass of the insulation for a given heat input. The heat-conduction equation and the optimality equation were solved numerically to obtain optimum density profiles for various values of thickness of the insulation. It is shown that for a given thickness using functionally graded foams can reduce the mass of the insulation panel.

## Acknowledgments

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# Multiblock Compressible Navier–Stokes Flow Solver Applied to Complex Launch Vehicles

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## Introduction

THE development of computational-fluid-dynamics procedures has progressed rapidly during the past two decades. Simultaneously, the rapid development in computer hardware has not only matched the explosive algorithm development but has indeed provided, and continues to provide, its impetus. Together, the computational resources are now available for the numerical simulation of the flow about many complex three-dimensional aerospace configurations. An efficient and accurate flow solver is the key to developing a useful engineering tool for the analysis of complex three-dimensional flow phenomena about complex configurations. Consequently, there is an avid interest in finding solution methodologies that will produce results in less time and cost, compared

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