Effect of Thermal Stresses on the Failure Criteria of Fiber Composites

Martin Leong*
Institute of Mechanical Engineering
Aalborg University, Aalborg, Denmark

Bhavani V. Sankar†
Department of Mechanical and Aerospace Engineering
University of Florida, PO Box 116250
Gainesville, FL 32611-6250, USA

Abstract: When composite laminates are operated at cryogenic temperatures, thermal stresses arise. This is due to the difference in coefficients of thermal expansion of different plies and also between the fiber and matrix. While the former is taken into account in the composite structural analysis, the latter, called micro-thermal stresses, has not been given much attention. In this paper the Direct Micromechanics Method is used to investigate the effects of micro-thermal stresses on the failure envelope of composites. Using FEA the unit-cell of the composite is analyzed. Assuming the failure criteria for the fiber and matrix are known, the exact failure envelope is developed. Using the micromechanics results, Tsai-Wu failure envelope is modified to account for the micro-thermal stresses. The approach is demonstrated using two example structures at cryogenic temperature.

KEY WORDS: composite failure criteria, cryogenic temperature, Direct Micromechanics Method, fiber composites, finite element method, micromechanics, thermal stresses

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* Graduate Student
† Ebaugh Professor, Author to whom correspondence should be addressed, Email: Sankar@ufl.edu

Introduction

As composite structures are more commonly operated at temperatures different from their stress free reference temperature, e.g., cryogenic tanks [1], the need for accurate thermal analysis procedures arises. Yet, when these procedures involve phenomenological failure criteria, the thermal stresses are only dealt with on a macroscopic level or ply-level. The mismatch of the coefficients of thermal expansion (CTE) of the fiber and matrix materials is also a source of thermal stress, which is not accounted in stress analysis, as the composite materials are usually modeled as homogeneous orthotropic materials. Consider a change in temperature ΔT in a unidirectional composite modeled as an orthotropic material. If the lamina is allowed to expand freely, then the strains will be given by $\alpha_1 \Delta T$, etc., and there will be no stresses in the composite. Then, any of the phenomenological failure criteria such as maximum stress, Tsai-Hill or Tsai-Wu theories would not predict failure of the composite, as there are no stresses. However, there will be stresses within the composite because of the mismatch in CTEs of the fiber and matrix. The average of these stresses, which are the macro-stresses, will be equal to zero. However, if the micro-thermal stresses are large enough, they can cause failure of the matrix or the fiber-matrix interface. Such failures cannot be predicted by the aforementioned phenomenological failure criteria.

In this paper the Direct Micromechanics Method (DMM) is used to investigate the effect of micro-thermal stresses on the failure of fiber reinforced composites at cryogenic temperatures. These results are compared to results obtained using available phenomenological failure criteria. The DMM, first proposed by Sankar, is a finite element-based micromechanical analysis of the composite unit cell (also called a representative volume element). It has been used in several articles, e.g., [1-5], to analyze and evaluate phenomenological failure criteria. The DMM can be thought of as a numerical laboratory, capable of simulating a variety of loading combinations, which may be difficult to achieve in the actual laboratory. This can range from uni-axial stress states, to full 3D stress states. The DMM procedure can be divided into two parts - a finite element stress analysis of the unit-cell and a micromechanical failure analysis (MFA). In this paper we suggest modifications to phenomenological criteria by taking into account the apparent loss of strength at cryogenic temperatures. Then, the modified failure criteria, which are adjusted for the micro-thermal stresses, are found to be satisfactory in predicting failure in composite structures operated at cryogenic temperatures. The procedures are illustrated by considering two example composite structures.

Micro-Mechanical Analysis

The goal of the finite element-based micro-mechanics is to obtain the unit-cell response, when subjected to different loads, either mechanical or thermal. In the DMM procedure, the unit-cell is subjected to six different independent macro-strains and one thermal load case. In practice this results in seven different FE models, which however share geometry, material properties, elements and mesh properties as explained in the following paragraphs.

The unit cell was analyzed using the commercial software Abaqus®, Version 6.6-1. In this paper, two different coordinate systems will be used. The 123-system refers to the principal material directions, with the 1-axis coinciding with the fiber direction. The *xyz*-coordinate system is used in the FEA model to apply boundary conditions etc. The *z*-axis coincides with the fiber direction. The two coordinate systems are shown in Figure 1.

A hexagonal unit-cell is used in this study (Figure 2). The hexagonal geometry is chosen over the square as the hexagonal pattern is closer to the random fiber distribution commonly found in unidirectional fiber composites [1]. The characteristic length *L*, fiber volume fraction V_f, fiber radius *r* and thickness *t* of the unit cell are presented in Table 1.The material properties chosen are typical of carbon/epoxy composite, since this is commonly used material in aerospace structures for cryogenic applications. The material properties are given in Tables 2 and 3. The tensile, compressive and shear strengths are denoted by S_T, S_C and S_{12}, respectively. It should be noted that the purpose of this paper is not to accurately determine the properties of a carbon/epoxy composite, but to compare the DMM with other failure criteria. As such, the values of the material properties are of less important.

As mentioned earlier, the unit-cell is subjected to six different mechanical load cases, and one thermal load case, which will be explained in a following section. In each of the mechanical load cases, the six macro-strains of the composite are set to unity one at a time. The periodic boundary conditions for the six unit-strain cases and the thermal case are presented in Table 4. This part of the FEA is based upon work done by Stamblewski, et al. [5]. In Table 4 *u*, *v* and *w* denote the displacements in the coordinate directions. The subscripts a_0, a_1, etc. denote the different sides of the hexagonal unit cell as shown in Fig. 2. The symbol *L* denotes the distance between the opposite faces of the hexagon.

The stiffness or elasticity matrix [*C*] of the homogenized composite can be calculated from the macro-stresses in the unit-cell for each of the unit macro-strain cases as

$$C_{ij} = \frac{\sigma_i^{(j)}}{\varepsilon_j^{(j)}} \quad , \qquad i,j = 1,\ldots,6 \ \text{(no summation)} \qquad (1)$$

In the above equation $\sigma_i^{(j)}$ is the i^{th} macro-stress for the j^{th} unit-strain case and $\varepsilon_j^{(j)} = 1$. The six macro-stresses for each unit strain case are calculated as the volume average of the corresponding micro-stresses:

$$\sigma_i = \frac{1}{V}\sum_{e}^{e_{\max}} \sigma_i^{(e)} V^{(e)} \quad , \qquad i = 1,\ldots,6 \qquad (2)$$

where $V^{(e)}$ and *V*, respectively, are the volumes of element *e* and the unit cell, $\sigma^{(e)}$ is the average micro stresses in element *e* and $e_{\max}$ is the total number of elements. For the present example, the elasticity matrix of the composite was found to be

$$[C]=\begin{bmatrix}161 & 3.8 & 3.8 & 0 & 0 & 0\\ & 10.4 & 4.6 & 0 & 0 & 0\\ & & 10.4 & 0 & 0 & 0\\ & SYM & & 3.9 & 0 & 0\\ & & & & 3.9 & 0\\ & & & & & 3.3\end{bmatrix}\text{GPa} \tag{3}$$

From [C], the engineering elastic constants (see table 5) are calculated using the relationship

$$\boldsymbol{C}^{-1}=\begin{bmatrix}1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0\\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0\\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0\\ 0 & 0 & 0 & 1/G_{23} & 0 & 0\\ 0 & 0 & 0 & 0 & 1/G_{31} & 0\\ 0 & 0 & 0 & 0 & 0 & 1/G_{12}\end{bmatrix} \tag{4}$$

The results shown in Table 5 are compared with that from Rule of Mixtures (RoM) just to make sure there are no major errors in the FE analysis.

The thermal load case (seventh case) consists of applying boundary conditions on the unit cell such that all macro-strains are equal to zero, and subjecting the entire unit cell to a uniform temperature change ΔT . Usually ΔT is taken as 1 C. The coefficients of thermal expansion can be calculated using the relation [3]

$$\{\alpha\}=-\frac{1}{\Delta T}[C]^{-1}\{\sigma\}^{(7)} \tag{5}$$

where $\{\sigma\}^{(7)}$ are the macro-stresses corresponding to the thermal case (seventh load case). For the present example problem α's are found to be

$$\begin{Bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{Bmatrix}=\begin{Bmatrix}-0.16\\ 26.7\\ 26.7\end{Bmatrix}\times 10^{-6}/\text{C} \tag{6}$$

Table 6 shows a comparison between the above CTEs and the values calculated using the rules of mixtures (RoM) [6]. It can be seen that the relative error between the FEA and the RoM results for α_1 is somewhat larger than that for α_2. However, it should be noted that both values of α_1 are close to zero, and there will be no significant thermal strains in the fiber direction.

Micromechanical Failure Analysis

The objective of the micromechanical failure analysis (MFA) is to use the results of micromechanical stress analysis in order to predict failure initiation in the composite. Let us consider a composite laminate subjected to force resultants $\left(N_x, N_y, N_{xy}\right)$, moment resultants $\left(M_x, M_y, M_{xy}\right)$, and temperature differential ΔT. Then, we can use the micromechanics to determine the micro-stresses in each of the element used in the finite element analysis of the unit-cell. Since we know the failure criteria of the fiber and matrix materials and also the interface, we can determine if failure has initiated in any of the elements. Of course, failure of one element cannot be considered as the ultimate failure of the composite. However, it is similar to first-ply failure in laminates, and then the forces acting on the laminate correspond to failure initiation.

In this work we use the maximum stress failure criteria for the fiber and matrix materials. The fiber-matrix interface failure is based on the interface tensile stress and the interfacial shear stress [2]. We assume if the interface is under compression, it has no effects on interfacial failure.

Let the force and moment resultants in the laminate be represented by $\{N\}$ and $\{M\}$. The mid-plane strains and curvatures in the composite can be calculated using the classical lamination theory as [6]

$$\begin{Bmatrix} \varepsilon_0 \\ \kappa \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} \begin{Bmatrix} N + N_T \\ M + M_T \end{Bmatrix} \tag{7}$$

where the laminate stiffness is defined by the so called *A*, *B* and *D* matrices, N_T and M_T, respectively, are the thermal forces and moments. Then, the strains in a ply of interest in the laminate at a location *z* is obtained as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \{\varepsilon_0\} + z\{\kappa\} \tag{8}$$

The strains can be transformed to obtain the strains $[\varepsilon_1\ \varepsilon_2\ \gamma_2\ ]^T$ in the principal material directions. The stresses in the ply are then derived as

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [Q]\left\{ \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{Bmatrix} \right\}$$
$$\sigma_3 = \tau_{31} = \tau_{32} = 0 \text{ (plane stress)} \tag{9}$$

We will assume the transverse shear strains γ_{13} and γ_{23} in the ply are negligibly small. The extensional strain ε_3 is calculated from the plane stress assumption as

$$\varepsilon_3 = -\frac{\nu_{13}}{E_1}\sigma_1 - \frac{\nu_{23}}{E_2}\sigma_2 \tag{10}$$

Having calculated all the six (macro) strain components and temperature in the ply, we can apply the micromechanical failure analysis (MFA) to check if the ply failure has occurred or not. Let the macro strains in the ply be represented as a 6×1 column matrix $\{\varepsilon^M\}$, where the superscript M denotes that these are macro-strains. Then the micro-stresses $\{\sigma^\mu\}$ in a finite element, say Element e, of the unit-cell can be obtained by superposition as

$$\underset{(6\times1)}{\{\sigma^\mu\}^{(e)}} = \underset{(6\times6)}{[H]^{(e)}}\underset{(6\times1)}{\{\varepsilon^M\}} + \Delta T \underset{(6\times1)}{\{s\}^{(e)}} \tag{11}$$

where the matrix $[H]$ is the matrix of influence coefficients. For example, the first column of H contains the six stresses in Element e for a unit-macro strain ε_1 in the composite. The six columns of H correspond to the six unit-stain cases of DMM discussed in the preceding section. The column matrix $\{s\}^{(e)}$ contains the stresses in Element e for a unit ΔT (seventh case of DMM).

Once we know the state of stress in an element, we can determine its failure status from its failure criterion. This procedure is repeated for all the elements including the fiber-matrix interface in the unit-cell FE model. A flow chart describing the aforementioned procedures is presented in Fig. 3. Details of the DMM failure analysis have been given in various papers, e.g., [2-5]

DMM Failure Envelope

The micromechanical failure analysis procedures described above can be used to develop failure envelopes for the composite. In this case we start from the macro-stresses in the ply rather than from force and moment resultants. For example, the failure envelope in the $\sigma_1 - \sigma_2$ plane can be developed by varying the values of these two macro-stress components and determining various combinations that correspond to failure initiation in the composite. Such a failure envelope is shown in Fig. 4. In this figure the symbols denote the DMM failure envelope.

From the DMM failure envelope one can develop phenomenological failure criteria. For example, the Tsai-Wu failure criterion under plane stress conditions is given by

$$F_{11}\sigma_1^2 + F_{22}\sigma_1^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 = 1 \tag{12}$$

The Fs in the above equation are strength coefficients that can be expressed in terms of uniaxial strengths as follows [6]:

$$F_1 = \frac{1}{S_{1T}} - \frac{1}{S_{1C}},\ F_{11} = \frac{1}{S_{1T} \cdot S_{1C}}$$
$$F_2 = \frac{1}{S_{2T}} - \frac{1}{S_{2C}},\ F_{22} = \frac{1}{S_{2T} \cdot S_{2C}} \tag{13}$$
$$F_{66} = \frac{1}{S_{12}^2}$$

where S_{1T} and S_{1C} are the tensile and compressive strengths in the 1-direction (fiber direction), S_{2T} and S_{2C} are the strengths in the 2-direction (transverse directions) and S_{12} is the shear strength. It should be noted that we have not included the coupling term $F_{12}\sigma_1\sigma_2$ on the LHS of Eq. (12). We found a better fit to DMM results without the coefficient F_{12}. Narayanaswami and Adelman [7] have also made the same observation in fitting the experimental results to Tsai-Wu failure envelope. We can estimate the strengths S_{1T}, S_{1C}, S_{2T} and S_{2C} using the DMM, and then use them in the above equations (Eqs. 12 and 13) to plot the phenomenological failure envelope. It should be noted that we have replaced physical testing in the laboratory by the simulations on the unit-cell in order to determine the strength values of the composite. For the example considered herein, the Tsai-Wu envelope is shown by solid line in Fig. 4. One can note that there are areas wherein the Tsai-Wu is conservative compared to DMM and there are areas where it overestimates the strength. The comparison between the two failure envelopes is given in Table 7 in terms of percentage of areas where the Tsai-Wu over predicts the strength determined by DMM. One can note that the Tsai-Wu criterion over-predicts (un-conservative) 4% of the cases and under-predicts (conservative) 12% of the cases. The areas were computed from the figure using the software SolidWorks.

Effects of thermal stresses on the failure envelope

The aforementioned procedures were repeated for the case ΔT = -80 C. The DMM failure envelope is shown in Fig. 5. In the same figure we have shown the Tsai-Wu failure envelope also. One can note that the DMM envelope is smaller indicating there is an apparent loss of strength. In fact the tensile strength in the 1-direction has significantly reduced (Table 8). Similar observations were also made by Whitley and Gates [8]. Note the deviation between the DMM at -80 C and Tsai-Wu is significant as indicated in Table 7 (third row of Table 7). At ΔT=-80 C the T-W theory over-predicts the strength 26% of the time. The reduction in strength is due to micro-thermal stresses that develop, especially in the matrix phase. As we will demonstrate in the next section, these effects could not be captured by the laminate level thermal stress analysis. We suggest modifying the Tsai-Wu failure envelope with the strengths measured at -80 C. The uniaxial strengths at ΔT=-80 C were obtained form the DMM analysis. The strength values are given in Table 8. Using these strengths one can plot the modified T-W envelope, which is shown in Fig. 6. One can note that the modified T-W envelope, which is adjusted for micro-thermal stresses, fit the DMM data well. It over-predicts strength only 4% of the time (last row of Table 7). To further illustrate the practical application of modified Tsai-Wu criterion we present two illustrations in the next section.

Illustration of Modified Tsai-Wu Criterion

We illustrate the application of modified Tsai-Wu criterion in two example composite structures. Example 1 is a pressure vessel subjected to both hoop stresses and longitudinal stresses. Example 2 is a laminate subjected to bending moments M_x and M_y. We calculate the maximum loads that can be applied before failure at room temperature (ΔT=0) and at a cryogenic temperature (ΔT=-80 C). For both plates Classical Lamination Theory (CLT) is used to find the ply stresses. These stresses are then input to the DMM procedure and the Tsai-Wu failure criterion to obtain the maximum load that can be applied according to each method. We tacitly assume that there are no residual thermal stresses at room temperature.

Example 1

Consider a thin-walled composite pressure vessel with closed end caps. The force resultants are given by $N_x=pD/4$, $N_y=pD/2$ and $N_{xy}=0$, where D, the mean diameter, is taken as 1 m. Our goal is to determine the maximum allowable pressure p using both DMM and Tsai-Wu failure criterion. We consider four different symmetric laminates as listed in Table 9. The strength coefficients listed in Table 8 are used in determining the maximum pressure that can be applied according to T-W criterion. This is a straightforward textbook problem [6]. In applying the DMM we chose an iterative approach where the pressure p is increased from a low value until failure is noticed in one of the finite elements in the micromechanical model of the unit-cell (see flowchart in Fig. 3). We performed the calculations for both room temperature (ΔT=0) and a cryogenic temperature (ΔT=-80 C).

The results are listed in Table 9 for ΔT=0. From Table 9 one can note that the Tsai-Wu theory predicts the maximum pressure quite well. Actually, the predictions are conservative (safety factor >1) and the difference is within 5%. On the other hand at cryogenic temperature (3rd column of Table 10) the T-W predictions are non-conservative (safety factor <1) and the deviation from DMM are in the range of 22 - 34%. This is due to the fact that the classical laminate analysis takes into account the thermal stresses that arise due to mismatch in the CTEs of the plies, but it does not account for the micro-thermal stresses. However, the micro-thermal stresses are accounted for by the modified Tsai-Wu criterion using the adjusted strength coefficients. The maximum pressures calculated using the modified T-W criterion are shown in the last column of Table 10. Still the results are non-conservative; however the deviation from DMM results is much less.

Example 2

In the previous example the laminate was subjected to in-plane forces only. In the second example we consider the same laminates subjected to bending moments $Mx=M_y=M_0$, and we determine the maximum M_0 before failure using both DMM and Tsai-Wu theory. As before we consider two cases ΔT=0 and ΔT=-80 C. The results are summarized in Table 11 for ΔT =0 and in Table 12 for ΔT =-80 C. The results are very similar to that for Example 1. The T-W criterion works well at room temperature (see Table 11) although the results for M_{0max} are slightly un-conservative. Again at ΔT =-80 C

(Table 12) we have to use the modified T-W criterion to predict the maximum bending moment accurately.

Conclusions

The Direct Micromechanics Method (DMM) is a powerful tool to determine if a composite laminate can withstand a given set of force and moment resultants at a given temperature. However, this is an expensive method, as each element in the finite element model of the unit-cell has to be analyzed for failure. On the other hand, phenomenological failure criteria such as Tsai-Wu criterion can be derived from the DMM, and can be efficiently used. Traditional thermo-mechanical stress analysis of composite structures account for thermal stresses at ply-level that arise due to difference in CTEs of the plies. However, there are thermal stresses at micro-level due to mismatch in CTEs of fiber and matrix materials, which are not accounted for in the structural mechanics. This is because of the fact that unidirectional composites are treated as homogeneous materials. The micro-thermal stresses can be significant at cryogenic temperatures where large stresses develop in the matrix phase due to CTE mismatch. The DMM, which is the analysis of the fiber and matrix elements in the unit-cell, includes the micro-thermal stresses automatically, and thus provides a more accurate failure analysis. On the other hand, the Tsai-Wu failure envelope can be modified to account for the micro-thermal stresses and can be used in conjunction with the traditional laminate thermal stress analysis. The proposed method was illustrated in two composite structures to determine the allowable loads. The modified Tsai-Wu criterion was able to predict the maximum loads with good accuracy compared to DMM.

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