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**Abstract** The success of an engineering project typically involves multiple stakeholders beyond the designer alone, such as customers, regulators, or design competitors. Each of these stakeholders is a dynamic decision maker, optimizing their decisions in order to maximize their own profits. However, traditional design optimization often does not account for these interactions, or relies on approximations of stakeholder preferences. Utilizing game theory, we propose a framework for understanding the types of interactions that may take place and their effect on the design optimization formulation. These effects can be considered as an economic uncertainty that arises due to limited information about interactions between stakeholders. This framework is demonstrated for a simple example of interactions between an aircraft designer and an airline. It is found that even in the case of very simple interactions, changes in market conditions can have a significant impact on stakeholder behaviors and therefore on the optimal design. This suggests that these interactions should be given consideration during design optimization.

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## **1** Introduction

Many modern engineered systems involve multiple stakeholders, each providing some inputs and receiving some outputs with respect to the system. In the simplest case, this might be a designer who determines system characteristics and a customer who determines how to utilize the system. In more complex systems, we might also have system operators, regulators, or suppliers. We may additionally have multiple stakeholders within each of these groups competing with one another, for example multiple designers each providing similar products to their customers. Each of these stakeholders acts as a dynamic decision maker, acting and reacting based on the decisions made by other stakeholders. These types of interactions can have a dramatic effect on the success or failure of a design.

There are several methods designers currently use to attempt to understand these interactions, mostly by attempting to uncover the preferences of other stakeholders. Most frequently, designers use legacy information based on the types of designs they and their competitors have produced before and the success of those designs. A designer may also use direct communication with other stakeholders, such as via a market study, to attempt to determine the relative importance of different performance metrics. However, these methods are not exact, and the resulting understanding of stakeholder preferences will have some error. This may be due to sampling bias of legacy designs, extrapolation into a new design space, or in cases of direct communication, miscommunication of preferences, either through a stakeholder's ignorance of their own preferences or a deliberate attempt to sway the designers' decisions. We can consider these errors in understanding stakeholder preferences as an economic uncertainty, directly changing a designer's true objective function and therefore affecting the design optimization process.

In order to understand the effects of these stakeholder interactions, we can utilize game theory [1]. Game theory has been developed in economics as a way to model strategic decision making between rational stakeholders, or players. Depending on the way players interact and the information shared between them, we can arrive at different outcomes for the same basic design problem. From the perspective of our optimization problem, game theory allows us to adaptively update our objective function, relating the performance characteristics of our design to designer profits, based on our location in the design space, changes in the market, and actions of other stakeholders. We will introduce this idea in more detail with some simple examples in the next section.

Previous works such as Vincent [2], Rao [3], Badhrinath and Rao [4], and Lewis and Mistree [5] have demonstrated the use of game theory for solving multidisciplinary design problems, but have not addressed the application of game theory to economic uncertainty and interactions. Li and Azarm study the design of a product [6] or product family [7] in the presence of competitive products in the market and uncertain customer preferences, but do not model customers or competitors as dynamic decision makers. Subrahmanyam [8] also considers the idea of market uncertainties as affecting design optimality, but these uncertainties are taken as given

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values and are not affected by design decisions. Morrison [9] applies game theory to a case study of fuel efficiency innovation among competing airlines, but does not consider additional stakeholders or applications to design optimization. The present work also draws from the ideas of decision based design [10] and value driven design [11] as tools for explaining design value as a function of performance attributes. The objective of this work is to reformulate a multidisciplinary design optimization problem to account for dynamic interactions between multiple stakeholders and market changes using a game theory model with both simultaneous and sequential interactions considered. We will additionally demonstrate, using an example from the aerospace industry, why considering these interactions during design optimization is important, and how it provides a designer with more information about design trade-offs.

The remaining part of the work is organized as follows. In Sect. 2 we provide our method of reformulating an optimization problem to account for different types of stakeholder interactions. In Sect. 3, we apply this method to a simple example problem of interactions between aircraft designers and regulators. Section 4 summarizes our conclusions, some limitations of the proposed framework, and plans for future work.

## **2** Problem Formulation

For the purpose of this work, we will focus on how we can reformulate an optimization problem when considering the effects of the interactions between l stakeholders. Readers interested in the principles of game theory can find more information from introductory game theory text books such as Fudenberg and Tirole [1]. First, let us consider a basic multidisciplinary design optimization problem formulation:

$$maximize \sum_{i=1}^{n} w_i f_i(\mathbf{X})$$

$$s.t.g_j(\mathbf{X}) \ge 0 \text{ for } j = 1, \dots, m$$

$$(1)$$

where **X** is our vector of design variables,  $f_i$  describes the *i*th performance metric of the design,  $w_i$  is the weight of the *i*th performance metric in the optimization, and  $g_i$  describes the *j*th of *m* many design constraints

By varying the vector w in this optimization, we can calculate a set of Pareto optimal designs for different performance values. Now consider that for each design and set of performance values (that is, each weight vector  $\mathbf{w}$ ) we can define some profit function for our designer,

$$\mathbf{\Pi}_1(\mathbf{w}, \mathbf{Y}, \mathbf{E}) \tag{2}$$

where **Y** describes the decision vector of the other stakeholders in the design and *E* describes a set of exogenous variables not directly controlled by any stakeholders. This function is used to transform our design performance and other stakeholder decisions directly into the profit for the designer. Note that our designer is labeled as the first stakeholder ( $\mathbf{Y}_1 = \mathbf{w}$ ) and there are l - 1 other stakeholders.

The decision vector  $\mathbf{Y}$  will be determined by the other stakeholders attempting to maximize their own expected profits, such that

$$\mathbf{Y}_{\mathbf{k}} = \operatorname{argmax}\left(\mathbf{\Pi}_{k}(\mathbf{Y}_{k}, \mathbf{w}, \mathbf{Y}_{\sim \mathbf{k}}, \mathbf{E})\right) \text{ for } \mathbf{k} = 2, \dots, l$$
(3)

where  $\Pi_k$  describes the profit of the *k*th stakeholder,  $\mathbf{Y}_k$  is the decision vector of the *k*th out of *l* many stakeholders,  $\mathbf{Y}_{\sim k}$  and is the decision vector of the other l - 2 stakeholders.

We now have l profit functions and l decision sets. This can be thought of as l different optimization problems, each dependent on the same decision vector for all players, forming an overdetermined set of equations. In order to determine a solution, we must apply a set of rules; in our case this is based on a certain game structure that describes the amount of information shared between stakeholders and the order in which decisions are made. Information shared between stakeholders refers to how well each stakeholder is able to approximate the profit functions of the others. For example, a designer may not explicitly know the profit function of their customer, but may make an approximation based on prior designs. We will also show that there may arise situations where one stakeholder may have an incentive to deliberately mislead another stakeholder in order to create a more favorable situation for themselves. This type of behavior need not be detrimental for the stakeholder being misled, and can in some cases be advantageous for both parties.

The order of decisions may be either simultaneous, sequential, or partially both. Sequential decision making means one stakeholder chooses their decision vector first and passes that decision on to the next stakeholder in the sequence. Stakeholders moving first will approximate the reaction of each subsequent stakeholder based on their available information about those stakeholders' profit functions. These approximated reactions are known as a best reply function [1]; that is, given that stakeholder one chooses  $Y_1$ , stakeholder 2 will maximize their expected profit by playing  $Y_2$ , or simply

$$\mathbf{Y}_i = \boldsymbol{\varphi}_{ij}(\mathbf{Y}_j, \hat{Y}) \tag{4}$$

where  $\varphi_{ij}$  is the best reply function that relates the given  $Y_j$  to the best reply  $Y_i$  and  $\hat{Y}$  is the vector of decisions of all the other stakeholders, some of which may be known based on the sequence of the game, and others which require their own best reply function to determine. Each of these can be solved recursively to determine a best reply function for each subsequent decision maker.

We can therefore formulate our profit maximization problem for the designer by combining Eqs. (1), (2), and (4), where the decisions of stakeholder acting in

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sequence before the designer are given as inputs, and the best reply function for stakeholders acting after the designer act as constraints. This problem will be subject to uncertainty in the exogenous inputs, **E**, as well as uncertainty due to approximations made in determining the best reply function,  $\varphi$ .

$$maximize \Pi_{1}(\mathbf{w}, \mathbf{Y}, \mathbf{E})$$

$$(\mathbf{X}) = \arg \max \sum_{i=1}^{n} w_{i} f_{i}(\mathbf{X})$$

$$(5)$$

$$\mathbf{x}, \mathbf{g}_{j}(\mathbf{X}) \geq \mathbf{0} \text{ for } \mathbf{j} = \mathbf{1}, \dots, \mathbf{m}$$

$$\mathbf{X}_{\mathbf{k}} = \varphi_{k1}(\mathbf{w}, \mathbf{Y}_{\sim k}) \text{ for } \mathbf{k} = 2, \dots, l$$

In the case of simultaneous decisions, we must use the concept of a Nash equilibrium [1] to determine a solution. A Nash equilibrium is a point in the decision space where no stakeholder can improve their own profit function by changing their decision vector. This means that a Nash equilibrium acts as a self-enforcing agreement between the players. That is to say, (X, Y) is a Nash equilibrium if and only if

$$\Pi_{1}(\mathbf{w}, \mathbf{Y}, \mathbf{E}) > \Pi_{1}(\mathbf{w}^{*}, \mathbf{Y}, \mathbf{E}) \text{ for all } \mathbf{w}^{*} \neq \mathbf{w}, \text{ and}$$
(6)  
$$\Pi_{k}(\mathbf{Y}_{\mathbf{k}}, \mathbf{w}, \mathbf{Y}_{\sim \mathbf{k}}, \mathbf{E}) > \Pi_{k}(\mathbf{Y}_{\mathbf{k}}^{*}, \mathbf{w}, \mathbf{Y}_{\sim \mathbf{k}}, \mathbf{E}) \text{ for all } \mathbf{Y}_{\mathbf{k}}^{*} \neq \mathbf{Y}_{\mathbf{k}}, k = 2, \dots, l$$

We can find any pure strategy Nash equilibria by formulating a best reply function for each stakeholder and solving that system of equations to determine where all the best replies intersect. A pure strategy Nash equilibrium means a stakeholder plays a single deterministic decision vector, while a mixed strategy means a stakeholder randomly selects from multiple pure strategies with some predetermined probability of each. It should be noted that there is no guarantee of a single unique Nash equilibrium, and equilibria can exist in both pure and mixed strategies. To solve our problem using simultaneous decision making, we are no longer performing an optimization. Instead, we are looking for the intersection of the surfaces defined by the best reply functions for each of our stakeholders. These intersections represent pure strategy equilibria, of which there may be multiple or none. In cases of multiple Nash equilibria, we can sometimes eliminate some equilibria through so called refinements. For the purposes of this work, we will present all Nash equilibria as possible outcomes, and we will only deal with simultaneous decision making in the discrete decision context for simplicity.

#### **3** Example Problem

Having defined how we may formulate an optimization problem considering interactions with other stakeholders, let us consider a simple example. We have two stakeholders, an aircraft designer and builder and their customer the airline. Both are monopolists, meaning they face no competition. We assume that the designer leases aircraft to the airline at a per flight cost that is fixed, regardless of the aircraft design or the number of flights.

The designer's only decision variable is the level of technology to invest in the aircraft, T. This can be thought of as the design effort and material and labor cost associated with producing the aircraft. For our problem, we will consider to be bounded between 0 and 1. T acts as the only weighting variable w as described in Eq. (1), where a value of 0 is the optimal manufacturing cost, and a value of 1 is the optimal customer value.

The airline's decision variable is the number of flights that they will offer, Q, which will determine the price they charge per ticket based on a fixed linear demand for air travel. The airline has some fixed cost of operation per flight, some cost that is proportional to the price of jet fuel,  $c_F$ , and some benefit based on the level of technology invested in the aircraft. We can then formulate the profit functions for both stakeholders as follows

$$\Pi_d(T, Q) = Q(L - c_T T) \tag{7}$$

$$\Pi_a(T, Q, c_F) = Q(P(Q)N_p - c_F F - c_L L + v_T T)$$
(8)

where  $c_T$  is the cost to implement new technology for the designer, F is the fuel consumption per flight, L is the lease cost per flight,  $c_L$  is some factor greater than 1 describing the total fixed costs for the airline including lease cost,  $v_T$  is the value of technology to the airline,  $N_p$  is the number of passengers per flight, and P(Q) is the price per ticket based on the linear demand function, given by

$$P(Q) = a - bQN_p \tag{9}$$

To create a meaningful example, we first find some reasonable estimates for some of the unknown coefficients in our problem. We select a Boeing 737-700 as the baseline aircraft for our analysis. Considering the standard configuration capacity of 128 passengers [12] and an average load factor of roughly 0.8 [13], we take the number of passengers per flight,  $N_p$ , as 100. Given an average flight length of 1000 miles [13], we calculate the fuel consumption per flight, F, as roughly 1500 gallons [14]. Average recent jet fuel prices are around \$3.00 per gallon [15], and we consider a range up to \$5.00 to account for possible future changes. Based on the 737-700 list price of \$76M [16] and a useful life of 60,000 flights [17] we find a per flight cost of \$1,300. Considering additional storage and maintenance costs as roughly doubling this expense, we select the per flight lease cost of the aircraft, L, as \$3000. Based on available airfare cost breakdown data [18], we consider that ranges  $c_L$  from 10 to 12, meaning that the capital cost of the aircraft ranges from 8 to 10% of the total cost per flight, depending on the airline. In order to determine characteristic numbers for the cost and value of new technology, we consider a new aircraft design project. We consider that this new design will cost an additional \$850 per flight, roughly a 25% increase from the initial design, and provides a benefit

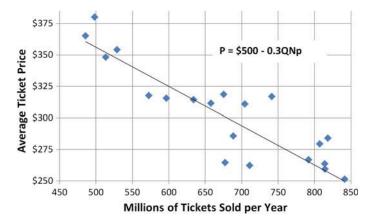


Fig. 1 Historical ticket price versus quantity sold [13, 19]

of \$4200 per flight through increased capacity, efficiency, and passenger comfort. Finally, by collecting data on tickets sold and average ticket price over the past 20 years, we fit the linear relationship between quantity and price as shown in Fig. 1. This approximation assumes that the airline uses this single aircraft design to service all of their routes.

Now let us consider the simplest case of interaction, where the designer first decides on the level of technology investment with full information about the airline profit function, and the airline then determines the quantity of flights in a sequential game. Note that both profit functions, Eqs. (7) and (8), are concave functions. We can therefore calculate a best reply function for the airline by setting to zero the first derivative of the airline profit function with respect to Q and solving for Q, such that

$$\frac{d\Pi_a}{dQ} = v_T T - c_L L - c_F F + N_p (a - bN_p Q) - N_p^2 Q b$$
(10)

$$Q^* = \varphi_{da}(T) = \frac{aN_p + v_T T - c_F F - c_L L}{2bN_p^2}$$
(11)

We can substitute this best reply function into the designer's profit function to replace and solve for the designer's optimal value of T by setting to zero the derivative of the designer's profit function with respect to T and solving for T,

$$\frac{d\Pi_d}{dT} = \frac{v_T(L - c_T T) + c_T(c_F F - c_L L - aN_p - v_T T)}{2N_p^2 b}$$
(12)

$$T^* = \frac{v_T L + c_F c_T F + c_L c_T L - a c_T N_p}{2 c_T v_T}$$
(13)

Using our values for our various coefficients, we can calculate the decision of the designer and airline and the profit for each. Since we have ranges of values for

c <sub>F</sub>	cL	$Q^*$	<b>T</b> *	$\Pi_d$	Па
\$3.00	10	2.58M	0 (-0.08)	\$7.75B	\$20.0B
\$3.00	12	2.02M	0.63	\$4.99B	\$12.32B
\$5.00	10	2.28M	0.27	\$6.30B	\$15.55B
\$5.00	12	1.78M	0.99	\$3.83B	\$9.57B

Table 1 Solution values for sequential game with no uncertainty

fuel price and the airline cost factor, we perform this analysis at the 4 extreme cases of these coefficients as shown in Table 1. Because our problem is linear in these values, we can interpolate between these 4 points to find the decisions and profits at any combination. Note that in the first case, the designer would choose an optimal value of slightly negative technology investment, however we restrict this value to be between 0 and 1. It can be seen that the optimal decisions and resulting profits for both the designer and airline vary greatly with these possible changes in parameters  $c_F$  and  $c_L$ .

In a realistic design problem, we will likely consider that a designer must make design decisions without knowledge of future fuel prices. These prices will be unknown to the airline as well. A designer will then maximize expected profits based on the possible distribution of future fuel prices. Due to the simple linear nature of our example problem, this will be the same as designing based on the mean value of future fuel prices.

A designer may face additional uncertainty in their understanding of the airlines' profit function, for example in the value of  $c_L$ . However, the airline will be able to know this value exactly. This is known in game theory as a game of "incomplete information" [1]. This means the designer will face some error in their prediction of the best reply function of the designer, specifically

$$Q^* = \varphi_{\tilde{d}a}(T) = \frac{aN_p + v_T T - c_F F - (c_L + \varepsilon)L}{2bN_p^2}$$
(14)

where  $\varepsilon$  describes the error in the designers understanding of airline costs.

We can see from our previous example that the designer will invest more in technology if they believe the airlines fixed cost,  $c_L$ , is higher. This is because higher fixed costs mean the effect of technology on airline marginal profits is more significant, and therefore more technology investment will have a greater effect on the quantity of flights. This relationship implies that airlines will have an incentive to mislead designers into believing that their costs are higher than in reality, shifting profits away from designers and toward airlines. Without considering the effects of these interactions, designers will be unable to understand the effects of these potential uncertainties.

To explore these interactions in more detail, let us switch from a continuous game to a discrete one. In this case, the designer must either decide to invest in new technology (T = 1) or not (T = 0). The airline will decide whether to expand their market by offering a higher number of flights (Q = 2.5M), or to maintain their

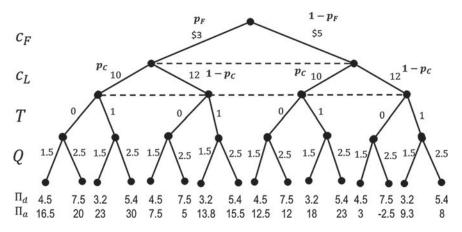


Fig. 2 Extensive form game with uncertainty in fuel prices p\_F and in fixed cost p\_C, where designers choose technology T and airlines choose quantity of flights Q with payoffs for the designer and the airline, respectively

current levels (Q = 1.5M). We consider that fuel prices will either be \$3 per gallon with probability  $p_F$  or \$5 per gallon with probability  $1 - p_F$ . Finally, the designer assumes the airline is a low cost carrier ( $c_L = 10$ ) with probability  $p_c$  or a high cost carrier ( $c_L = 12$ ) with probability  $1 - p_c$ . We can express this problem using a decision tree (cf. Fig. 2), known in game theory as an extensive form game [1].

In Fig. 2, each node represents a decision, and dashed lines between nodes indicate an information set, where the decision maker must act without knowing for certain which node in the information set they are currently in. The solution will therefore depend on the decision maker's beliefs about the values of  $p_F$  and  $p_c$ . The payoffs for each resulting set of decisions are given at the end of each path, where the top number is the designer's profit, and the bottom number is the airline's profit, both in billions of dollars. We can simplify this game by eliminating dominated strategies for the airline, since we know at the last branch of the decision tree the airline will choose the value that maximizes their own profits; this is known as backwards induction. Fig. 3 shows these dominated strategies in gray.

We see that, based on this discrete example, the designer can only influence the airline to utilize more flights by increasing technology investment if fuel prices are low and airline costs are high, or fuel prices are high and costs are low. In the remaining two cases, the designer will strictly prefer not to invest in new technology, since they will lease the same number of flights regardless and will have a higher profit margin for each. Airlines will always prefer the case where designers invest in technology, as they always gain higher profits.

From this simple example, we would conclude that if fuel prices are high, airlines will attempt to convince designers that they have low costs, as designers will believe they can then influence flight quantity by investing in technology. If fuel prices are low, airlines will attempt to convince designers that their costs are high, again in an effort to encourage designers to invest in technology.

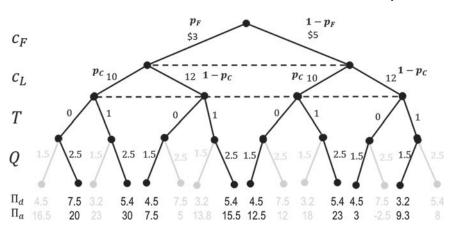


Fig. 3 Backwards induction indicating strictly dominated choices (gray) for the airline when choosing quantity Q

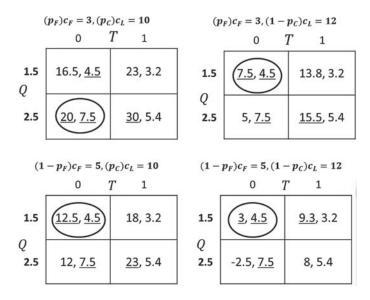


Fig. 4 Simultaneous game solution

We may also be interested to know if the possible solutions of this game change if we consider that designers and airline make decision simultaneously. For example, airlines submit orders for new aircraft without knowing future fuel prices or precise aircraft specifications. We can represent this sort of game using strategic form, with 4 payoff matrices representing the 4 possible combinations of fuel price and airline costs as shown in Fig. 4.

The numbers in each box represent the payoffs for the airline and the designer, respectively. Numbers that are underlined indicate a best reply for that stakeholder.

When both numbers are underlined in the same box, meaning the best replies intersect, we have a Nash equilibrium for that individual game, represented by circling that square. We can see that for the simple game we have constructed, it is never advantageous for the designer to invest in technology. This happens because since decisions are made at the same time, the designer's choice cannot influence the quantity selected by the airline. We can also see that when airline costs are high ( $c_L = 12$ ), meaning we are on the two matrices on the right side, the equilibrium solution for this game will be (T = 0), (Q = 1.5M). When airline costs are low, the equilibrium will depend on the probability of low fuel prices,  $p_F$ , as the airline will attempt to maximize their expected profits. If the airline believes  $p_F$  is less than 0.11, they will always choose the low quantity (Q = 1.5M), and if they believe  $p_F$  is greater than 0.11 the airline will choose the high quantity, (Q = 2.5M). When  $p_F$  is equal to 0.11, the airline is indifferent between these two strategies and may play either one, or play a mixed strategy where they randomly select between both options. It should be noted that the designer would strictly prefer the airline select the higher quantity, but based on this game structure, they have no way to influence that decision.

It should be noted that the solutions we have found for each of these different types of games need not be Pareto optimal in terms of profits for both stakeholders. For example, in Fig. 4, we can see that both the designer and a high cost airline  $(c_L = 12)$  would be strictly better off playing the strategy (T = 1), (Q = 2.5M) as compared to the equilibrium strategy (T = 0), (Q = 1.5M), regardless of the values of fuel price and airline costs. However, that strategy is not an equilibrium because one or both of the stakeholders can improve their profits by modifying their decision. For example, in the case of  $[c_F = 5, c_L = 12]$  starting at (T = 1), (Q = 2.5M), we see that the designer would strictly prefer to select (T = 0) when the airline plays (Q = 2.5M), and similarly the airline prefers (Q = 1.5M) against (T = 1). Because the strategies and payoffs are known, each player will realize the other will try to change their own strategy, and will respond accordingly, resulting in selecting (T = 0), (Q = 1.5M). This is a variation on the classical game theory example known as the prisoner's dilemma [1].

# 4 Conclusions

We have presented a framework for how game theory can be utilized in design optimization to better model and understand interactions between multiple stakeholders. We demonstrated how, based on the order in which interactions take place and the information shared between stakeholders, the optimal decision for the designer can change significantly. By incorporating these interactions into the design problem, we can directly anticipate these changes and can quantify the uncertainty in the profit expected for our final design based on approximations of other stakeholders. Additionally, this framework is able to directly provide information for the designer regarding trade-offs between multiple disciplines during design, since we are able to adaptively update the designer's objective function based on changes in stakeholder preferences due to changes in performance.

Using our simple example problem, we demonstrate that for the sequential game between the designer and airline, small changes in the value of certain profit function coefficients can have a large effect on optimal design choices and profits for both stakeholders. We observe that for the values we have selected in our sequential game, the airline may have an incentive to obscure their true costs from designers in order to encourage investment in new technology. Looking at the same problem but using a simultaneous structure, the designer will never elect to invest in technology, based on the cases considered. From these two examples we have shown that understanding the structure of the game can greatly change the outcome, and that, within that structure, approximations by one stakeholder in the preferences of another can have a large impact on design decisions and profits.

We do note that, depending on the game structure utilized, a stakeholder may need to approximate the decisions of the designer in their profit maximization, requiring them to solve the design optimization problem within their own profit optimization. For expensive design problems, this creates computational limitations, and future work is needed to address this issue. It can also be difficult in a practical problem to quantify the type of interactions between multiple stakeholders. The authors have previously proposed a method to understand these interactions by using causal models [20]. Future work in this area will focus on applying the methods described to a realistic design problem and understanding the relative importance of uncertainty in stakeholder preferences as compared to traditional design uncertainties like variations in material properties and operating conditions.

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