EAS6939 Homework #6

1. In engineering, the random wind pressure \( Y \) is typically modeled as a quadratic transformation of the random wind speed \( X \). If \( X \sim N(\mu, \sigma^2) \) and \( Y = X^2 \), find approximation of \( Y \) based on the first-order Taylor series expansion about mean of \( X \) and equivalent linearization. For equivalent linearization, consider \( Y_L = aX + b \), where \( a \) and \( b \) are optimal parameters. Calculate the mean and variance of the above approximations of \( Y \).

2. A vehicle has a deterministic mass, \( m = 2 \), and random velocity, \( V \), which can take on both positive and negative values. The kinetic energy \( K \) of the vehicle is \( K = \frac{1}{2} m V^2 \). If \( V \) follows Normal (Gaussian) probability distribution with mean, \( m_V = 0 \), and standard deviation, \( \sigma_V = 1 \), determine the probability density function and cumulative probability distribution function of \( K \). Use the method of general transformation.

3. The resistance (or strength), \( R \), of a mechanical component which is subject to a load, \( S \), are modeled as random variables with the following probability density function:

\[
\begin{align*}
    f_r(r) &= \begin{cases} 
      0.5 & 0 \leq r \leq 2 \\
      0 & \text{otherwise}
    \end{cases} \\
    f_s(s) &= \begin{cases} 
      2s & 0 \leq s \leq 1 \\
      0 & \text{otherwise}
    \end{cases}
\end{align*}
\]

Assume that \( R \) and \( S \) are statistically independent. Find the cumulative probability distribution functions, \( F_Y(y) \) and \( F_Z(z) \) of

(a) \( Y = R - S \)
(b) \( Z = R/S \)

Furthermore, evaluate
(c) \( F_Y(0) \)
(d) \( F_Z(1) \)
(e) Explain why \( F_Y(0) = F_Z(1) \)