Useful Motor/Torque Equations for EML2322L

<u>Force</u> (Newtons)

$F = m \times a$ m = mass (kg)

a = acceleration (m/s^2)

<u>Motor Torque</u> (Newton-meters)

$T = F \times d$ F = force (Newtons)

d = moment arm (meters)

<u>Power (Watts)</u>

$P = I \times V$ I = current (amps)

V = voltage (volts)

$P = T \times \omega$ T = torque (Newton-meters)

ω = angular velocity (radian/second)

<u>Unit Conversions</u>

Length (1 in = 0.0254 m)

Velocity (1 RPM = 0.105 rad/sec)

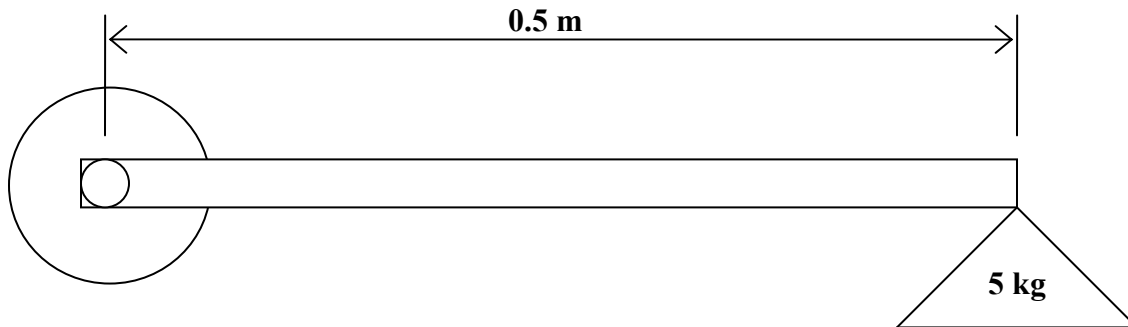Torque (1 in-lb = 0.112985 N-m)

Power (1 HP = 745.7 W)

Example 1

Determine if the following motor can be used to lift a 5-kg load using a 0.5-m lever arm.

Merkle-Korff Gearmotor specifications
Stall Torque = 40 in-lb
Stall Current = 3.5 amps

0.5 m

5 kg

Solution

Convert Stall Torque from in-lb to N-m
    1 in-lb = 0.112985 N-m
    40 in-lb = 40 x 0.112985 N-m = 4.5194 N-m

Calculate the Force required to lift the 5-kg load
    $F = m \times a = 5 \text{ kg} \times 9.81 \text{ m/s}^2 = 49.05 \text{ N}$

Calculate the Torque required to lift the Force with the lever arm
    $T = F \times d = 49.05 \text{ N} \times 0.5 \text{ m} = \boxed{24.525 \text{ N-m}}$

We cannot perform the lift with this set-up, because the stall torque is smaller than the torque required for the lift. We must either shorten the length of the lever arm, or we must choose another motor with a higher stall torque to perform this operation.

Example 2

Using the same motor as in Example 1 with a 12-V power supply:
a) Calculate the power used by the motor to rotate a 5-kg load at 50 RPM using a 3-inch lever arm.
b) Calculate the current draw from the battery to perform this operation.

Solution

Convert inches to meters
      1 in = 0.0254 m
      3 in = 0.0762 m

Calculate the Force required to lift the 5-kg load
      $F = m \times a = 5 \text{ kg} \times 9.81 \text{ m/s}^2 = 49.05 \text{ N}$

Calculate the Torque required for this operation
      $T = F \times d = 49.05 \text{ N} \times 0.0762 \text{ m} = 3.738 \text{ N-m}$

      Note- This toque is lower than the motor's stall torque, so this operation is possible using the specified motor, mass, and lever arm

Convert RPM to radians/second
      1 RPM x 2π rad/rev x 1 min/60 sec = 0.105 rad/sec
      ω = 50 rev/min x 0.105 rad/sec/RPM = 5.25 rad/sec

Calculate the Power required for this operation
      $P = T \times \omega = 3.738 \text{ N-m} \times 5.25 \text{ rad/sec} = \boxed{19.622 \text{ W}}$

Calculate the Current draw from the battery (use the supply voltage in this calculation)
      $I = P/V = 19.622 \text{ W}/12 \text{ V} = \boxed{1.635 \text{ Amps}}$

      Note- This current is smaller than the maximum allowable current draw of the motor.

Example 3

Determine the motor torque necessary to power the robot drive wheels.

Solution

The following approach is merely one way to solve this problem. Several exist.

Assume the robot will be powered by two powered drive wheels and supported by two freely rotating caster wheels. Robot weight is denoted by W and for this simple example, we'll assume the weight is distributed evenly over all 4 wheels, as shown in Figure 1 below.
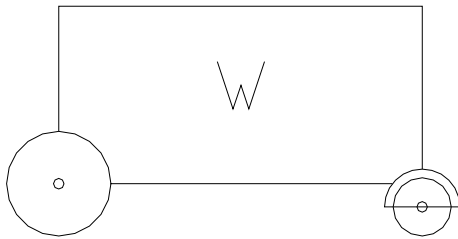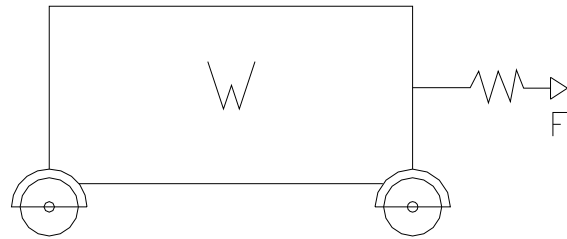

Figure 1   Figure 2

Thinking logically about the problem, we could model the robot as having 4 of the identical caster wheels (Figure 2) and the force required to propel the robot is simply the force needed to start the robot moving (this could be measured empirically with a force scale). The problem is we haven't yet built the robot so testing it in this manner is not an option; we need to calculate the force (and hence motor torque) required to move the robot **before** we build anything.

Looking closer at the caster wheel we can see the actual friction that must be overcome to put the robot in motion. F_w is the friction force between the wheel and the floor and F_a is the friction force between the wheel and the axle. T_w and T_a are the respective torques between the wheel and floor and the wheel and axle.
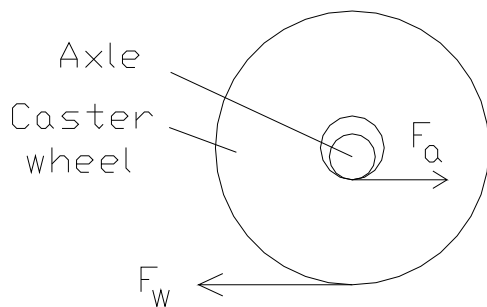

Figure 3

$$F_a = W/2 * \mu_a$$
$$T_a = F_a * R_a$$

$$F_w = W/2 * \mu_w$$
$$T_w = F_w * R_w$$

T_w is the *maximum* torque the wheel can transmit to the ground before it slips.

Our goal is to find a realistic range for T_m, the motor torque.

As calculated above, T_w would be the *maximum* amount of torque the motor could transfer to the ground before the wheel begins to slip (ie $T_{m, max}$).

Typically, we desire $\mu_w > \mu_a$, so the wheel does not slip/slide across the floor, but rather rolls. We can easily look up the μ_a value for the axle/wheel materials in contact. Knowing μ_a and the weight of the vehicle, F_a can be computed. This is the *minimum* amount of force we would have to provide at the wheel/axle interface to overcome the friction between the two. To relate the computed axle force F_a to the *minimum* amount of wheel torque required to move the robot, we would use the "virtual radius" of the wheel/axle combination, which is computed as follows:

$$R_v = R_w - R_a$$

This is the fictitious radius about which F_a would act to rotate the wheel about the tangent point in contact with the ground at any instant, as shown in Figure 4 below.
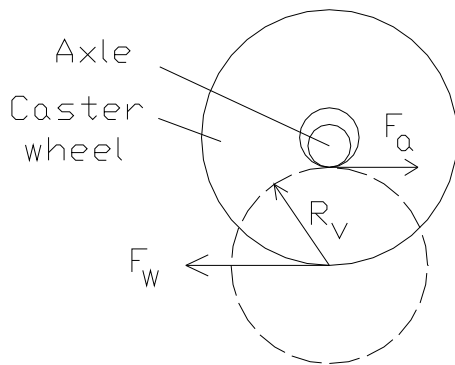

Figure 4

Therefore our equation for the *minimum* amount of torque the motor must transfer to the ground before the wheel begins to roll (thus causing the robot to move) would be:

$$T_{m,min} = F_a * R_v = F_a * (R_w - R_a)$$

In summation, $T_{m, min} \leq T_m \leq T_{m, max}$ or alternatively, $\mathbf{F_a * (R_w - R_a) \leq T_m \leq F_w * R_w}$