EAS6939 Homework #6

1. In engineering, the random wind pressure (Y) is typically modeled as a quadratic transformation of the random wind speed (X). If $X \sim N(\mu_x, \sigma_x^2)$ and $Y = X^2$, find approximation of Y based on the first-order Taylor series expansion about mean of X and equivalent linearization. For equivalent linearization, consider $Y_L = aX + b$, where a and b are optimal parameters. Calculate the mean and variance of the above approximations of Y.

2. A vehicle has a deterministic mass, $m = 2$, and random velocity, V, which can take on both positive and negative values. The kinetic energy (K) of the vehicle is $K = \frac{1}{2}mV^2$. If V follows Normal (Gaussian) probability distribution with mean, $m_V = 0$, and standard deviation, $\sigma_V = 1$, determine the probability density function and cumulative probability distribution function of K. Use the method of general transformation.

3. The resistance (or strength), R, of a mechanical component which is subject to a load, S, are modeled as random variables with the following probability density function:

$$f_R(r) = \begin{cases} 0.5 & 0 \le r \le 2 \\ 0 & \text{otherwise} \end{cases} \qquad f_S(s) = \begin{cases} 2s & 0 \le s \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Assume that R and S are statistically independent. Find the cumulative probability distribution functions, $F_Y(y)$ and $F_Z(z)$ of

(a) $Y = R - S$

(b) $Z = R/S$

Furthermore, evaluate

(c) $F_Y(0)$

(d) $F_Z(1)$

(e) Explain why $F_Y(0) = F_Z(1)$