$ORIGIN \equiv 1$ $\quad \Xi \equiv ORIGIN$

$MPa \equiv 10^6 Pa$ $\quad \mu\varepsilon \equiv 10^{-6}$

Problem 2.5:

$$T := \begin{pmatrix} 80 & 20 & 40 \\ 20 & 60 & 10 \\ 40 & 10 & 20 \end{pmatrix} MPa$$ Stress tensor at a point

a.) Find the stress vector on the plane described by the vector v:

$$v := \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$ Vector describing a plane

$$n := \frac{v}{|v|} \qquad n = \begin{pmatrix} 0.4082 \\ 0.8165 \\ 0.4082 \end{pmatrix}$$ Normal vector on the plane described by v

$$t_i = T_{ji} \cdot n_j \qquad t := T^T \cdot n \qquad t = \begin{pmatrix} 65.3197 \\ 61.2372 \\ 32.6599 \end{pmatrix} MPa$$ Stress vector on the plane described by v

b.) Determine the principal stresses σp:

$$\left| T_{ij} - \sigma_p \cdot \delta_{ij} \right| = \sigma_p^{\,3} - I_1 \cdot \sigma_p^{\,2} + I_2 \cdot \sigma_p - I_3 = 0$$ Eigenvalue problem

$$\sigma p := reverse(sort(eigenvals(T))) \qquad \sigma p = \begin{pmatrix} 110 \\ 50 \\ 0 \end{pmatrix} MPa$$ Principal stresses

$i := \Xi .. last(\sigma p)$ Indexing

c.) Find the maximum shear stress:

$$\tau max := \frac{1}{2} \cdot \left(\sigma p_1 - \sigma p_3 \right) \qquad \tau max = 55\,MPa$$ Maximum shear stress

d.) Find the octahedral shear stress:

$$I1 := \sum_{i=1}^{3} \sigma p_i \qquad I1 = 160\,MPa$$ First invariant

$$\sigma oct := \frac{1}{3} \cdot I1 \qquad \sigma oct = 53.3333\,MPa$$ Octahedral normal stress (hydrostatic pressure)

$$np_i := \frac{1}{\sqrt{3}} \qquad np = \begin{pmatrix} 0.5774 \\ 0.5774 \\ 0.5774 \end{pmatrix}$$ Normal on the octahedral plane

$$\tau oct := \left| \frac{\sigma p}{\sqrt{3}} - \sigma oct \cdot np \right| \qquad \tau oct = 44.9691\,MPa$$ Octahedral shear stress

Problem 2.56:

a.) Determine the displacement and strain fields for the x-y coordinate system:

$u(x,y) = a_1 + a_2 \cdot x + a_3 \cdot y + a_4 \cdot x \cdot y$ $\quad u(0,0) = 0mm \quad u(1,0) = -2mm$

$u(1,1) = -5mm \quad u(0,1) = -3mm$

$v(x,y) = b_1 + b_2 \cdot x + b_3 \cdot y + b_4 \cdot x \cdot y$ $\quad v(0,0) = 0mm \quad v(1,0) = 1mm$

$v(1,1) = 3.5mm \quad v(0,1) = 2.5mm$

$uv := \begin{pmatrix} 0 \\ -2 \\ -5 \\ -3 \end{pmatrix} mm$ Nodal x-displacement vector

$vv := \begin{pmatrix} 0 \\ 1 \\ 3.5 \\ 2.5 \end{pmatrix} mm$ Nodal y-displacement vector

$X := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} m$ Matrix of x-y coordinate values

$a := \text{lsolve}\left(\frac{X}{m}, \frac{uv}{m}\right)$ $\quad a = \begin{pmatrix} 0 \\ -0.002 \\ -0.003 \\ 0 \end{pmatrix}$ X-displacement coefficients

$b := \text{lsolve}\left(\frac{X}{m}, \frac{vv}{m}\right)$ $\quad b = \begin{pmatrix} 0 \\ 0.001 \\ 0.0025 \\ 0 \end{pmatrix}$ Y-displacement coefficients

$u(x,y) := a_1 + a_2 \cdot x + a_3 \cdot y + a_4 \cdot x \cdot y$ X-displacement field $\quad u(x,y) \text{ float}, 2 \rightarrow (-2.) \cdot \frac{mm}{m} \cdot x - 3. \cdot \frac{mm}{m} \cdot y$

$v(x,y) := b_1 + b_2 \cdot x + b_3 \cdot y + b_4 \cdot x \cdot y$ Y-displacement field $\quad v(x,y) \text{ float}, 2 \rightarrow \frac{mm}{m} \cdot x + 2.5 \cdot \frac{mm}{m} \cdot y$

$\varepsilon_{ij} = \frac{1}{2} \cdot \left(u_{i,j} + u_{j,i}\right)$ Strain-displacement relationship

$\varepsilon xx = u_{x,x} = -0.002 \qquad \varepsilon yy = v_{y,y} = 0.0025$

$\varepsilon xy = \frac{1}{2} \cdot \left(u_{x,y} + v_{y,x}\right) = \frac{1}{2} \cdot (-3 + 1) = -0.001$

$\varepsilon xx := -0.002$ X-normal strain

$\varepsilon yy := 0.0025$ Y-normal strain

$\varepsilon xy := -0.001$ XY-shear strain

b.) Determine the strain field for the X-Y coordinate system:

$\varepsilon := \begin{pmatrix} \varepsilon xx & \varepsilon xy \\ \varepsilon xy & \varepsilon yy \end{pmatrix}$ $\varepsilon = \begin{pmatrix} -0.002 & -0.001 \\ -0.001 & 0.0025 \end{pmatrix}$ Strain tensor in xy coordinate system

$\theta := 30deg$ Angle from x-axis to X-axis

$A := \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$ $A = \begin{pmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{pmatrix}$ Transformation matrix

$E := A^T \cdot \varepsilon \cdot A$ $E = \begin{pmatrix} -1741.0254 & 1448.5572 \\ 1448.5572 & 2241.0254 \end{pmatrix} \mu\varepsilon$ Strain tensor in XY coordinate system

Problem 2.67:

$\varepsilon xx = A\cdot(L - x)$ $\varepsilon yy = B\cdot(L - x)$ $\varepsilon xy = 0$ Strain field

$u(0,0) = v(0,0) = 0$ $u(0,0)_{x,y} = v(0,0)_{y,x}$ Boundary conditions

Integrate for the displacement fields

$u(x,y) = \int \varepsilon xx\, dx = A\left(L\cdot x - \frac{1}{2}\cdot x^2\right) + Y(y)$ $v(x,y) = \int \varepsilon yy\, dy = B\cdot y\cdot(L - x) + X(x)$

$\varepsilon xy = \frac{1}{2}\cdot\left(u_{x,y} + v_{y,x}\right) = \frac{1}{2}\cdot\left(Y_{y,y} + X_{x,x} - B\cdot y\right) = 0$ Shear strain

$Y_{y,y} - B\cdot y = -X_{x,x} = K$ Rearranging terms and..

$X = \int -K\, dx = -K\cdot x + C$ $Y = \int B\cdot y + K\, dy = \frac{1}{2}\cdot B\cdot y^2 + Ky + D$ Integrating

$u(x,y) = \frac{-1}{2}\cdot A\cdot x^2 + \frac{1}{2}\cdot B\cdot y^2 + A\cdot L\cdot x + K\cdot y + D$ $v(x,y) = -K\cdot x + B\cdot L\cdot y - B\cdot x\cdot y + C$

where C, D, and K are unknown constants

Given that $u(0,0) = v(0,0) = 0$ $D = 0$ and $C = 0$

Given that $u(0,0)_{x,y} = v(0,0)_{y,x}$ $B\cdot y + K = -B\cdot y - K$ $K = -K$ $K = 0$

$u(x,y) = A\cdot\left(L\cdot x - \frac{1}{2}\cdot x^2\right) + \frac{1}{2}\cdot B\cdot y^2$ $v(x,y) = B\cdot y\cdot(L - x)$ Displacement fields

Problem 2.71:

$u(x,y,z) = c_1 \cdot x \cdot z$ $\quad v(x,y,z) = c_2 \cdot y \cdot z$ $\quad w(x,y,z) = c_3 \cdot z$ Displacement field

$\varepsilon_{ij} = \frac{1}{2} \cdot \left(u_{i,j} + v_{j,i}\right)$ $\quad \varepsilon(x,y,z) = \begin{pmatrix} c_1 \cdot z & 0 & 0.5 \cdot c_1 \cdot x \\ 0 & c_2 \cdot z & 0.5 \cdot c_2 \cdot y \\ 0.5 \cdot c_1 \cdot x & 0.5 \cdot c_2 \cdot y & c_3 \end{pmatrix}$ Strain field

a.) Strain at point E:

$$\varepsilon(c_1, c_2, c_3) := \begin{pmatrix} 2 \cdot c_1 & 0 & 0.75 \cdot c_1 \\ 0 & 2 \cdot c_2 & 0.5 \cdot c_2 \\ 0.75 \cdot c_1 & 0.5 \cdot c_2 & c_3 \end{pmatrix}$$ Strain at point E

b.) Normal strain at point E in the direction of line EC:

$\tan(\theta) = \frac{E_x}{E_z} = \frac{1.5}{2}$ $\quad \cos(\theta) = \frac{2}{2.5}$ $\quad \sin(\theta) = \frac{1.5}{2.5}$

$A := \frac{1}{5} \cdot \begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}$ Transformation matrix $\quad \varepsilon pxz(c_1, c_3) := \begin{pmatrix} 2 \cdot c_1 & 0.75 \cdot c_1 \\ 0.75 \cdot c_1 & c_3 \end{pmatrix}$ Strain on xz-plane

$A^T \cdot \varepsilon pxz(c_1, c_3) \cdot A \text{ float}, 3 \rightarrow \begin{pmatrix} .560 \cdot c_1 + .360 \cdot c_3 & 1.17 \cdot c_1 - .480 \cdot c_3 \\ 1.17 \cdot c_1 - .480 \cdot c_3 & 1.44 \cdot c_1 + .640 \cdot c_3 \end{pmatrix}$ Strain rotated into the XZ-system

$\varepsilon EC(c_1, c_2, c_3) := 1.44 \cdot c_1 + 0.64 \cdot c_3$ EC lies along the Z axis

c.) Shear strain at E between EF and ED:

$\varepsilon xy := 0$ Strain at E is zero, because the xy-strain is 0, and EF-ED lies in the xy-plane

d.) Determine the strain values for the following coefficients:

$c := \begin{pmatrix} 0.002 \\ 0.004 \\ -0.004 \end{pmatrix}$ Coeficients

$\varepsilon(c_1, c_2, c_3) = \begin{pmatrix} 4000 & 0 & 1500 \\ 0 & 8000 & 2000 \\ 1500 & 2000 & -4000 \end{pmatrix} \mu\varepsilon$ Strain at point E

$\varepsilon EC(c_1, c_2, c_3) = 320 \mu\varepsilon$ Normal strain at point E in the direction of line EC

$\varepsilon xy = 0$ Shear strain at E between EF and ED