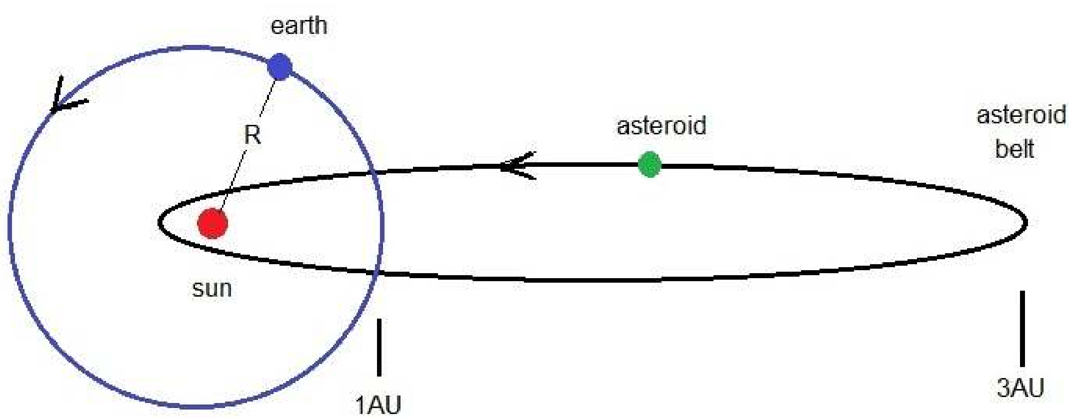
IMPACT SPEED OF AN ASTEROID WITH THE EARTH

Yesterday(Feb.15, 2013) the earth had a close call with a football field sized asteroid passing within 17,000 miles of the earth's surface. About the same time a smaller asteroid, estimated at 15 meter diameter, made a direct collision with the earth near Chelyabinsk, Russia. Although most experts claim these were unrelated events, a simple consideration of earth-asteroid collision probabilities suggests the events are related much like the movement of shotgun pellets. Be that as it may, we want here to determine the approximate collision velocity of an asteroid whose origin is the asteroid belt located at approximately three astronomical units from the sun. Recall that 1AU=14.9597 x 10^7 km is the approximate radius R of the earth's nearly circular orbit about the sun. A schematic of the problem under consideration is the following-

ORBITS OF THE EARTH AND AN ASTEROID ON A POSSIBLE COLLISION COURSE



We are assuming the sun, earth and asteroid orbits all lie in the same plane, that the earth's orbit is essentially a circle of radius R= 1AU while the asteroid of mass m follows a highly elliptical orbit with an apogee of 3AU. We designate the asteroids speed at R=1AU as V and at the asteroid belt distance equal to the slower speed V_b. From the conservation of energy one has-

$$\frac{1}{2}mV^2-\frac{GMm}{R}=\frac{1}{2}mV_b^{\,2}-\frac{GMm}{3R}$$

Here G is the universal gravitational constant and M the mass of the sun. We can replace GM by use of the force balance-

$$\frac{GMm_e}{R^2} = \frac{m_e V_e^{\ 2}}{R} = \frac{4\pi^2 R\ m_e}{\tau^2}$$

where m_e and V_e are the earth's mass and speed in its circular orbit about the sun. Also τ=60x60x24x365.25=3.15576 x 10^7 sec are the number of seconds in a year. Solving for the asteroid collision speed V , we find-

$$V = \left[V_b^{\ 2} + \frac{16\pi^2 R^2}{3\tau^2} \right]^{1/2}$$

To get an estimate for the speed V_b of the asteroid at the far point of its orbit we assume this equals the overall speed of the asteroid belt about the sun. We know this value from Kepler's Third Law which states that the square of an orbit period τ_b is proportional to the cube of the semi-major axis 3R of its orbit. One thus has that τ_b=sqrt(27)τ. From it we deduce that-

$$V = \left[(\frac{6\pi}{\sqrt{27}})^2 + (\frac{4\pi}{\sqrt{3}})^2 \right]^{1/2} (\frac{R}{\tau}) = [13.1594 + 52.6378]^{1/2}\ 4.7403\, km/\sec$$

That is, if an asteroid coming from the asteroid belt collides with the earth, the impact speed will be about –

V=8.1115 x 4.7403 km/sec=38.45 km/sec=23.89 miles/sec

This number is quite a bit larger than the escape velocity of 11.2 km/sec from the earth's surface but less than the sun's escape velocity at R=1AU of 42.1km/sec.The largest known asteroid is the 950km diameter dwarf-planet Ceres. Fortunately its abhelion of 2.98 AU and perihelion at 2.54 AU puts it in the asteroid belt with little chance of escaping.

The kinetic energy carried by an asteroid equals KE=(1/2)mV^2 which is huge when the mass gets large. The 1908 Tunguska collision in Siberia involved an asteroid of about 30 meters in diameter and the Chicxulub collision near the Yucatan some 65 million years ago was produced by an asteroid believed to have a diameter of 10 km . Some claim that it may have led to the demise of the dinosaurs. Fortunately such events are extremely rare and the chance of a large asteroid hitting a particular city is essentially zero. A much more dangerous event would be for a large asteroid to hit the ocean where it would create a huge tsunami which could drown millions living near the shoreline.

There has been much discussion in the news about what could be done in the event that an asteroid of 1km diameter or larger would be detected heading for a collision with earth. The best option would be to precisely determine its trajectory and then evacuate everyone within several hundred kilometers from the impact point.

Hollywood scenarios such as deflecting the asteroid via nuclear explosions are unrealistic because of the available time involved and also the miniscule momentum change a nuclear explosion would have on a fast moving asteroid of that size. Also fragmenting an asteroid, if this could be achieved, would likely be counter-productive as it will have a shotgun effect of distributing the single impact to multiple impact points over a much wider range not to mention wide nuclear contamination.