ON A PASCAL TRIANGLE WITH BUILT-IN ASYMMETRY

The standard Pascal triangle consists of an array of numbers as indicated-

PASCAL TRIANGLE

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                  1
               1     1
            1     2     1
         1     3     3     1
      1     4     6     4     1
   1     5    10    10     5     1
1     6    15    20    15     6     1
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This mathematical construct, dating back to the ancient Chinese, has its elements defined by-

C[n,m]=C[n-1,m-1]+C[n-1,m]

where n refers to the row and m the column number. Note the sum of the elements in the nth row is 2^n. In an even more compact form, one can express any element of Pascal’s Triangle by the binomial coefficient-

$$C[n,m]=\frac{n!}{m!(n-m)!}$$

with both n and m running from 0 through n. The number 20 in row 6 and column 3 is thus given by $6!/(3!^2)$. Note the perfect symmetry about the vertical line containing 1-2-6-20. One can also look at the Pascal Triangle as a probability measure for certain events such as flipping a coin to see if heads(H) or tails(T) will occur . On the first flip the chance of heads or tails are 50% each. To get two heads in a row has a 25% chance and to get three heads in a row has only a 12.5% chance of happening. These percentages can be deduced by looking at the fraction –

$$\frac{C[n,0]}{\sum_{m=0}^{n} C[n,m]}=2^{-n}$$

So after flipping a coin four times , the odds are HHHH=1, HHHT=4, HHTT=6, HTTT=4, and TTTT=1 . It says that the chance of having four flips in a row yield four heads is just 1 in 16. Professional gamblers and speculators are very much aware of this fact.

We want now to consider a variation on the Pascal Triangle involving a non-symmetric expansion of its elements as shown-

$$\begin{array}{c} 1 \\ a \quad b \\ a^2 \quad 2ab \quad b^2 \\ a^3 \quad 3a^2b \quad 3ab^2 \quad b^3 \\ a^4 \quad 4a^3b \quad 6a^2b^2 \quad 4ab^3 \quad b^4 \end{array}$$

This triangle clearly looses its symmetry when a≠b. Also the nth row elements are generated by the binomial expansion of $(a+b)^n$. The coefficients are given by-

$$C[n,m,a,b] = \frac{n!}{m!(n-m)!} a^n b^{n-m}$$

The above coin flipping experiment was carried out with a fair coin. If now instead we use a weighted coin which has the chance of yielding heads at 60% and the remaining 40% for tails, we can use the above asymmetric Pascal Triangle modification to estimate the likelihood of getting 4 heads in a row. Here we have n=4, a=0.6 and b=0.4. So the chance becomes-

$$\frac{(0.6)^4}{\sum_{m=0}^{4} C[4,m,0.6,0.4]} = \frac{0.1296}{0.4978713600} = 0.260308..$$

while that of four tails is smaller by a factor of 16/81. Remember a fair coin predicts 1/16=0.0625. Gambling casinos operate with much smaller advantages for the house. In typical European Roulette the wheel has 18 red, 18 black and one green pocket. When playing red or black, which I have done several times at the casino in Monte Carlo, the chance of the ball landing on any one pocket is 1/37. The chance of landing on any red is18/37 =48.6486%. The chance of not landing on red is therefore 19/37=51.3513%. The house thus has the advantage of 1/37=2.7027% on the money wagered. In American Roulette the house advantage is higher since it uses two green pockets(0 and 00) so that the chance of hitting red decreases to 9/19=47.268%. In either forms of Roulette the house will always win in the long run.

Let us next apply the asymmetric Pascal Triangle to a game of 50-50 chance . Let's say we have N dollars total available for this gamble and are willing to wager a fraction f of ones fund per bet. After the first bet we will have N(1+f) after a win and N(1-f) after a loss. On the next bet we will wager f times these amounts to get the four possible results $N(1-f)^2$, N(1-f)(1+f), N(1-f)(1+f), and $N(1+f)^2$. Letting the initial fund be N=1 and denoting (1-f) by L and (1+f) by W, we can write the betting sequence as-

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & L & & W & & & \\
 & & L^2 & & 2LW & & W^2 & & \\
 & L^3 & & 3L^2W & & 3LW^2 & & W^3 & \\
L^4 & & 4L^3W & & 6L^2W^2 & & 4LW^3 & & W^4
\end{array}
$$

So after the first bet one has either a loss reducing the fund to L=1-f or an equally likely gain increasing the fund to W=1+f. For the second bet, again using fraction f of the new total fund, we get four possible outcomes. These are L^2, LW, LW, and W^2. Combining the two equal middle terms, we generate the n=2 row in the above asymmetric Pascal Triangle. Continuing on and combining equal results, we generate the triangle shown. It tells us that after three consecutive bets the chance of winning all three produces a gain of $(1+f)^3$ but the likelihood of doing so is only one in eight. The element $4LW^3$ in the fourth row means that one is on the fourth bet and has won three of them and lost one. The likelihood of hitting this combination is 1/4th. If one bets all of one's fund so that f=1, you will either have doubled the fund or been wiped out after the first bet. A professional gambler would never wager all his funds on one fifty-fifty bet. When I play Roulette, I will set aside a small amount of say 100 dollars and then bet using f=1/20 which is equivalent to a five dollar bet. This way I can play a long time without loosing the entire initial fund. Under this condition one very likely will have nearly as many wins as losses. In reality the house always has a built-in advantage and thus guarantees that a typical gambler will most likely loose in the long run. Las Vegas loves high-rollers since the house advantage in this case brings in large amounts of cash to the casino.