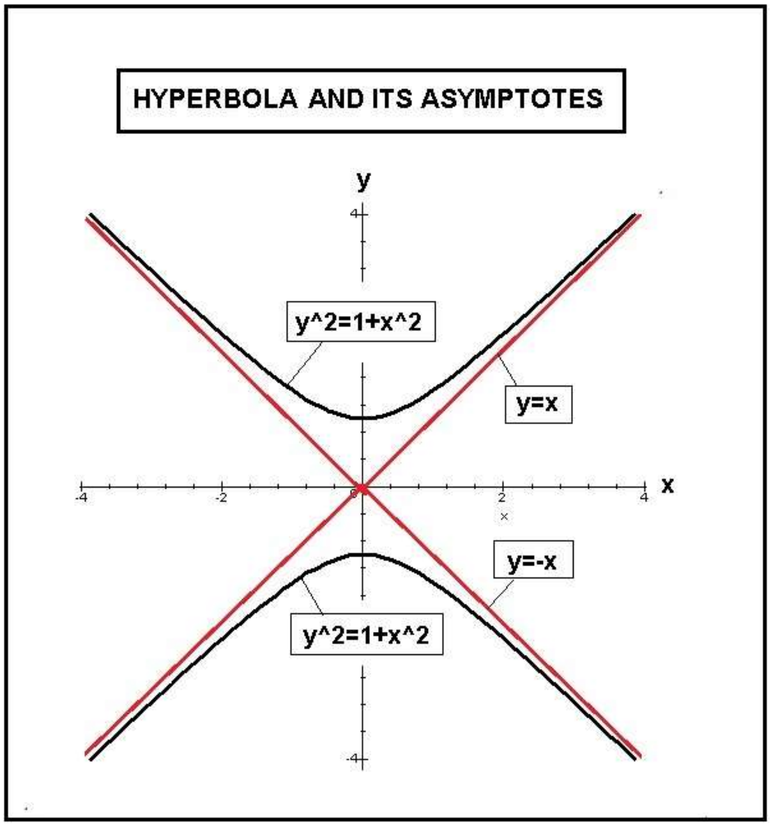
FINDING ASYMPTOTES FOR CONTINUOS FUNCTIONS

It is well known since ancient times that many functions possess asymptotes represented by linear relations of the form-

$$y=A + B x$$

, where A and B are fractions or whole integers unique to the function y(x) being considered. Historically it was the ancient Greek mathematician and astronomer Apollonius of Perga who around 200 BC first introduced the concept of an asymptote. He coined the word “asumptotos” meaning “not falling together”. As the meaning suggests, an asymptote is a straight line which approaches a curve y(x) closer and closer with changing x but never quite touches it. It was Apollonius’s studies on conic sections which obviously led to his asymptote discovery. Perhaps the best known asymptotes are the lines y=x and y=-x for the hyperbola y^2=x^2+1. Here is a graph of this curve (in black) and its two line asymptotes (in red)-



One sees that the asymptotes shown approach the hyperbola as $\pm x\ and\ \pm$y get large. They never completely touch.

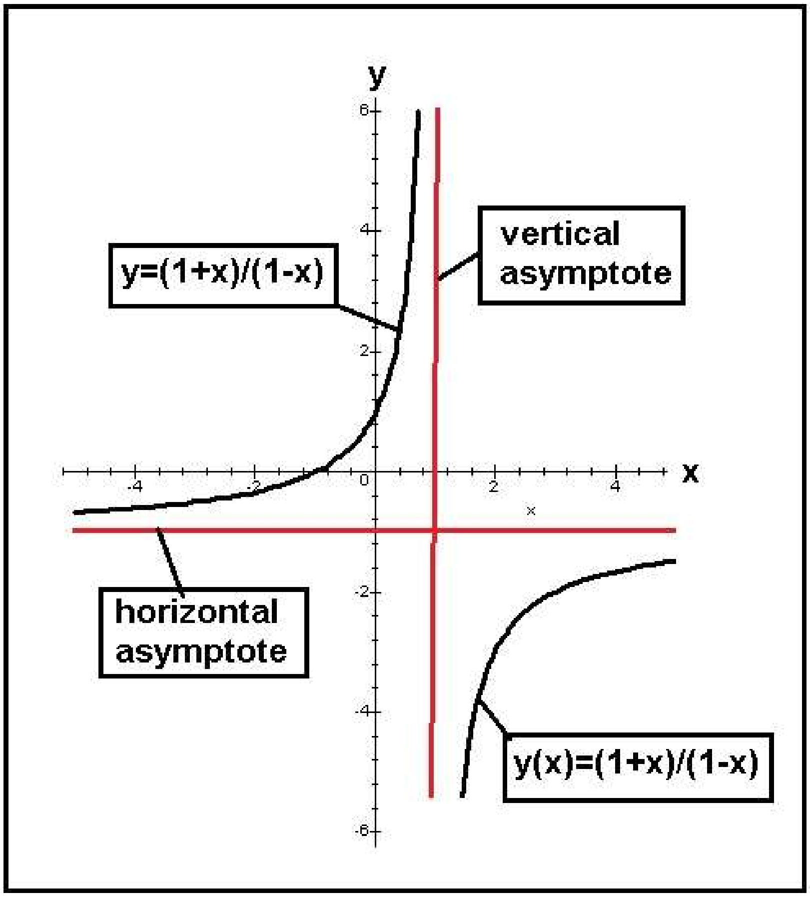
We want in this note to discuss the various types of possible asymptotes.

There are just three. These are the vertical, horizontal and oblique type depending upon the slope B. Lets look at several different cases.

We begin with the quotient function-

y(x)=(1+x)/(1-x)

This function is infinite at x=1 and has zero value at x=-1. It goes to minus one at x equal to plus or minus infinity. Here is its plot-



We see it has a vertical asymptote where the denominator of y(x) vanishes. The horizontal asymptote follows by letting the value of x in this quotient go to plus or minus infinity. Note if an asymptote comes in at any angle between 0 and π/2 radians the asymptote is referred to as oblique. The asymptotes involving the standard hyperbola as shown above come in at ±π/4 and hence are oblique.

It is not always necessary that one plots out the function y(x) in order to find its asymptotes. Take the case of-

$$y(x)=(x^2+5x-6)/(x^2-3x-4)$$

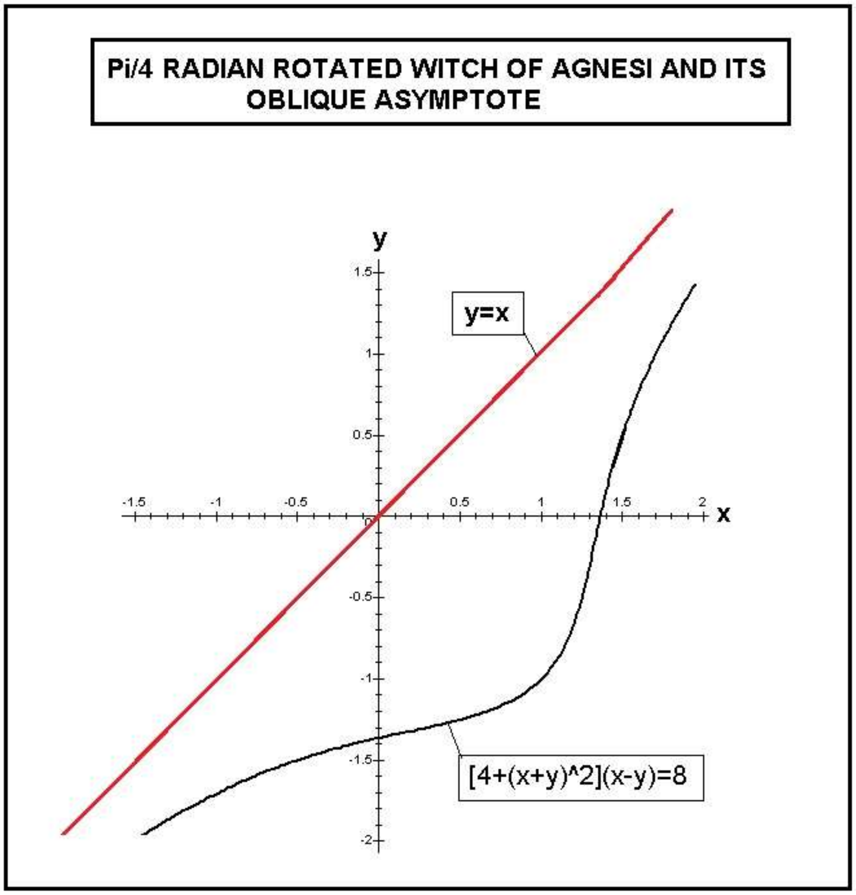
This has the obvious two vertical asymptotes at x=4 and x=-1. Also by letting x go to plus or minus infinity we have y(x) go to one. This means we also have a horizontal asymptote y(x)=1 for this function.

Some functions have an infinite number of asymptotes. These include y(x)=tan(x) which has infinite values at x=(π/2){2n+1} , with n being any positive or negative integer.

As a final example we look at a 45 degree rotated Witch of Agnesi curve. It has the rather complicted form-

$$(x-y)[4+(x+y)^2]=8$$

but the very simple oblique asymptote y=x. Here is its graph-



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