COOLING OF A CUP OF COFFEE

A well known problem to which many high school science students are exposed is to discover Newton's Law of Cooling by measuring the drop in temperature of a cup full of hot coffee. Most will indeed make the observation that the coffee temperature drops off exponentially and some even arrive at his law that-

$$\frac{dT}{dt} = -k(T - T_c)$$

, where k is a constant to be determined and Tc is the ambient temperature in the room where the temperature measurements are being made. The k encompasses all three modes of heat transfer- conduction, convection, and radiation- although the last will be of negligible importance at the sub-boiling temperatures typically encountered.

With a little calculus this first order differential equation may be solved as-

$$T(t) = T_c + (T_h - T_c)\exp(-kt)$$

, where t is the time and T_h the temperature of the coffee at the beginning of the measurements. Note that $T(0)=T_h$ and $T(\infty)=T_c$. To find k one usually carries out multiple measurements starting with different initial temperatures. To normalize these results it pays to introduce the non-dimensional version of the Newton Cooling Law-

$$\theta(t) = \frac{(T - T_c)}{(T_h - T_c)} = \exp(-kt)$$

k will also depend upon cups heat transfer properties and hence experiments must be conducted with the same cup under the same orientation.

It is our purpose here to repeat some measurements on coffee cooling described in the literature, add some date on the cooling of plain water, and to dispel the notion held by the public that pouring cream into hot coffee at the beginning keeps the mixture warm the longest.

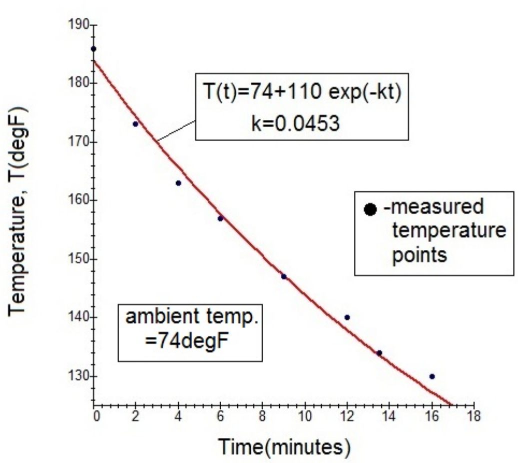
We began our measurements by making use of the following elementary experimental equipment-

EQUIPMENT FOR MEASURING THE COOLING OF A CUP OF COFFEE
(cylindrical cup,oven stick thermocouple 130 - 190degF, ruler, calculator)

The thermometer is a a stick thermocouple which my wife uses to check meat temperatures when cooking. Its range is 130degF to 190 degF. Typically warm drinkable coffee has a temperature range between 110 degF to 140 degF.

In our first experiment we used just plain water heated via microwave oven to about 190degF. The thermocouple was placed into the filled cup and its temperature was recorded as a function of time. The recorded raw data is summarized in the following graph-

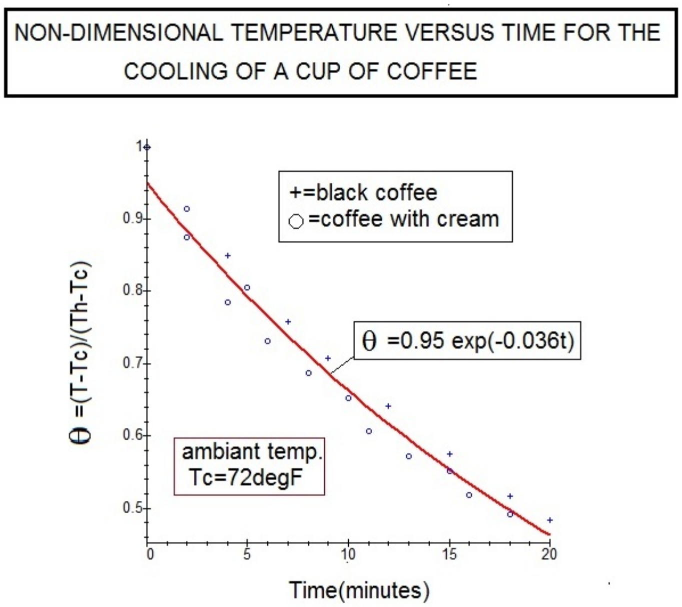
TEMPERATURE VERSUS TIME FOR A CUP OF HEATED WATER



The temperature indeed shows an exponential drop-off. The red curve is a slightly modified Newton Cooling Law with k=0.0453 and T_h-T_c=186-74=112 replaced by 110.

Our next excperiment involved black coffee heated to 190degF for the first reading and then to 184degF for a second reading. This time to normalize things we recorded the results in the non-dimensional form θ(t) versus t in minutes. The range θ(t) was from 1 down to about ½. This corresponded to a time of about 20 minutes.

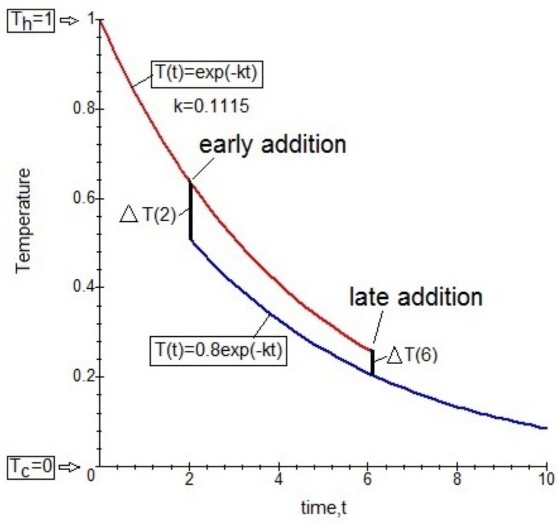
In our third set of experiments we added cream to the coffee and again heated things via microwave to about 192degF. The cream volumeV_c was approximately 10% of the coffee volume of V_h. Casting the results for black coffee and for coffee with added cream onto the same graph produces the following-



What is interesting about this result is that, whether or not cream is added to the coffee , the temperature variation in both cases has a similar exponential behavior and the two temperature drop offs follow at nearly the same exponential rate. This certainly proves that radiation can only play a very minor role in the cooling process since a black surface would be expected to cool faster than a white top surface and we have here the reverse. The red curve in the graph seems to also follow the Newton cooling Law with only slight differences.

We now come to the final point of this article. Why does the public believe that adding cream to hot coffee when it is first served will keep the mixture warm longer than when adding the cream at a later time? The present results show that this is not so. Look at the following hypothetical cooling diagram-

TEMPERATURE VERSUS TIME OF A CUP OF COFFEE BEFORE AND AFTER CREAM ADDITION



Here we have hot black coffee cooling until t=2 when cold cream is added. This drops the temperature by ΔT(2) and the drop off continues on a descending exponential curve shown in blue. The blue curve will always remain lower than the red curve for the same fixed time after cream addition. Should one wait to pour the cream until t=6, the resultant temperature drop ΔT(6)<ΔT(2) since the heat content of the coffee $Q_h= (\rho cVT)_h$is greater than$(\rho cVT)_c$. The cream volume V_c is typically small compared to the coffee volume and its initial temperature is cold compared to the coffee. If we take the product of the fluid density ρ and its specific heat c to be essentially the same for the two fluids, we find approximately that-

$$\Delta T(t) \approx (\frac{V_c}{V_h})T(t+\varepsilon)$$

in the limit as ε goes to zero. This implies that the temperature drop ΔT(t) is proportional to T(t) meaning that it gets smaller the larger t gets. The upshot is that the temperature of the mixture will always stay on the same blue curve after cream addition no matter when the cream is added. One can also reach the same conclusion by simply shifting the red curve backwards in time until it coincides with the blue curve.

To keep the coffee as hot as possible one must draw the obvious conclusion that good insulation, such as provided by a thermos bottle, is required. No matter what time a constant amount of cooler cream is added to the black coffee, the resultant mixture will always remain on a lower temperature curve relative to that of a cup of black coffee, in contrast to what some internet commentators believe.

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