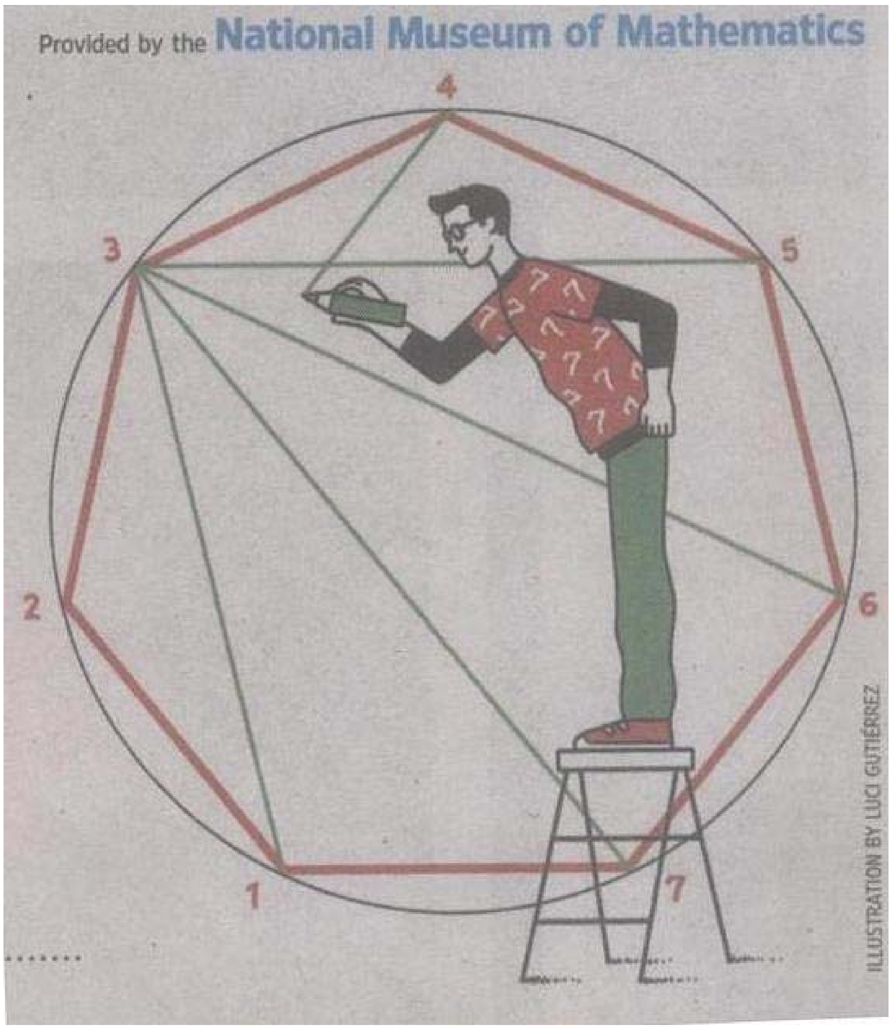
NUMBER OF UNIQUE DIAGONALS ONE CAN DRAW INSIDE A REGULAR POLYGON

In today's Wall Street Journal Puzzle Page of October 1, 2016 the question was asked how many unique diagonals one can draw inside a heptagon (ie-seven sides). The question is accompanied by the following picture-



This question is rather trivial as a visual inspection of the figure reveals. There are four diagonals which can be drawn from vertex 3. This is followed by four diagonals from vertex 4, follwed by just three diagonals from vertex 5, with just two from vertex 6 and one from vertex 7.The remaining vertex points 1 and 2 yield no additional diagonals not already present. Hence the answer is that the number of diagonals D equals-

$$D=4+4+3+2+1=14$$

A generalization , not mentioned in the article, is to make the problem a bit more challenging by discussing diagonals for n sided regular polygons. Doing so we arrive at the following table-

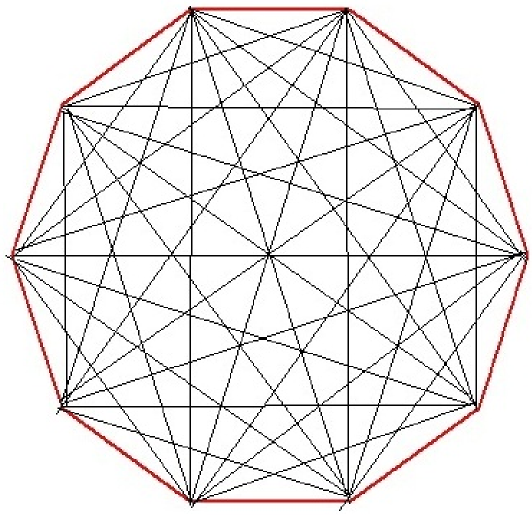
Side Number, n	Unique Diagonals, D	1^{st} Difference	2nd Difference
3 (Triangle)	0	-	-
4 (Square)	2	2	-
5 (Pentagon)	5=2+2+1	3	1
6 (Hexagon)	9=3+3+2+1	4	1
7 (Heptagon)	14=4+4+3+2+1	5	1
8 (Octagon)	20=5+5+4+3+2+1	6	1
9 (Nonagon)	27=6+6+5+4+3+2+1	7	1
10 (Decagon)	35=7+7+6+5+4+3+2+1	8	1

Noting that the second differences are all equal to one, suggests at once that D must go as a quadratic in n. Working out the constants for such an expansion , we arrive at the formula-

$$D = \left(\frac{n}{2}\right)[n-3]$$

which checks nicely with the numbers for D given in the table. A picture showing all thirty-five unique diagonals for a decagon follows-

DECAGON AND ITS THIRTY-FIVE DIAGONALS



diagonals=7+7+6+5+4+3+2+1=35

For a twenty sided polygon (icasagon) the number of unique diagonals will be –

D=(20/2)[20-3]=170

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