THE DODECAHEDRON

The five platonic solids have faces consisting of regular polygons. The simplest of these is the cube(hexahedron) with six faces in the form of squares and the most complex is the icosahedron with twenty faces in the form of equilateral triangles. The remaining three solids are the tetrahedron with four triangular faces, the octahedron with eight triangle faces, and finally the dodecahedron with twelve regular pentagons as faces. Having already looked at the icosahedron in some detail in one of our earlier discussions, let us now examine the regular dodecahedron more closely.

We begin with the following 3D plot generated with the MAPLE input-

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With(plots):
polyhedraplot([0,0,0],polytype=dodecahedron.scaling=CONSTRAINED,style=patch
, title='DODECAHEDRON);
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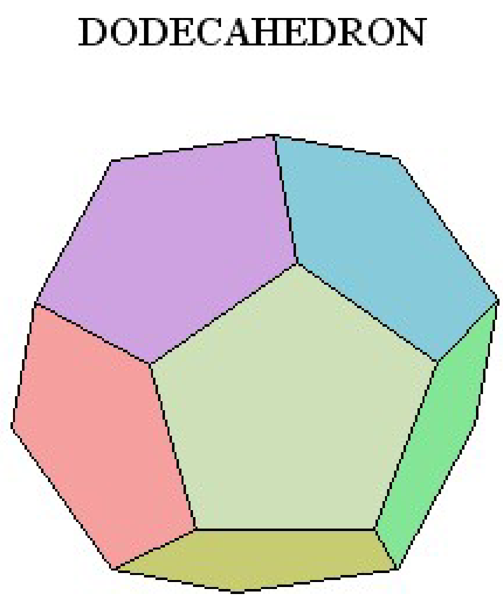


The edgelength of the 12 pentagon faces are assumed to be unity so that the total surface area of the dodecahedron equals-

$$A = 12(\frac{5}{4})\tan(54^{o}) = \frac{15\Phi}{\sqrt{3-\Phi}} = 20.6457288...$$

where Φ=[1+sqrt(5)]/2=1.6180339887… is the Golden Ratio.

To calculate the volume of the dodecahedron one must first determine the radial distance R from the volume center to any one of the twenty vertexes. This is accomplished by envisioning a sphere of the same radius with 20 equally spaced points representing the vertexes lying upon it. Taking the first point to lie at k(1,1,1) and placing a neighboring point at k(1/Φ,Φ,0) one finds the equal radial distances from the origin to the two vertexes to be R=ksqrt(Φ^{-2}+Φ^{2})=ksqrt(3) since Φ^{2}=Φ+1. Now since we are taking the side length of the pentagons to be unity, one has-

$$1 = k^2\left[(1-0)^2 + (1-\frac{1}{\Phi})^2 + (1-\Phi)^2\right] = \frac{(2k)^2}{1+\Phi}$$

Thus –

$$R = \frac{\sqrt{3}}{2}\sqrt{1+\Phi}$$

Next the distance from the center of a pentagon to the dodecahedron center will be-

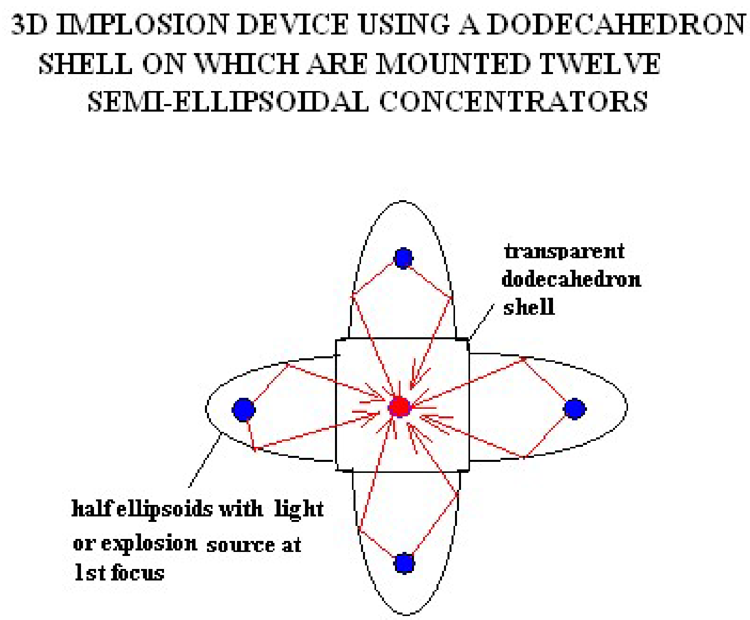
$$H = \sqrt{R^2 - (\frac{1}{3-\Phi})} = \frac{\sqrt{3\Phi+2}}{2\sqrt{3-\Phi}}$$

The volume of the dodecahedron will equal to the sum of the twelve pyramids whose height is H and base area is 5Φ /[4sqrt(3- Φ)]. One finds-

$$Vol = 12(\frac{1}{3})\frac{5}{4}\frac{\Phi}{\sqrt{3-\Phi}}\sqrt{\frac{3\Phi+2}{4(3-\Phi)}} = \frac{5}{2}\frac{\Phi\sqrt{3\Phi+2}}{(3-\Phi)}$$

$$= \frac{1}{4\sqrt{2}}(5+3\sqrt{5})\sqrt{7+3\sqrt{5}} = \frac{(15+7\sqrt{5})}{4} = 7.66311896...$$

In a wooden model of such a dodecahedron we have constructed in our workshop, we notice that the opposite surfaces are parallel to each other. This suggests an interesting design for an energy concentrator where one mounts semi-ellipsoids on each of the twelve pentagon surfaces as shown and has the ellipsoids second focal point located at the center of the dodecahedron. The configuration one envisages is as shown-



It is fairly easy to calculate the dimensions of the ellipsoids required. Their semi-minor axes will equal to half the diameter of the largest inscribed circle in a pentagon and thus becomes –

$$b = c = \frac{L}{2}\tan(54^o) = \frac{L\Phi}{2\sqrt{3-\Phi}} \quad where \quad L \;\; pentagon \quad side-length$$

The semi-major axis equals a=LH/ε where H is as given above and ε=sqrt(a^2-b^2)/a is the ellipse eccentricity. The shock compression of an object at the dodecahedron center achievable by the simultaneous ignition of twelve charges at the blue points should be considerable at the red point at time $\tau=2a/V_o$ later, where V_o is the shock propagation speed. Related devices are used as nuclear bomb triggers, kidney stone lithotripsy, and in attempts to produce nuclear fusion by compressing small deuterium pellets.

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