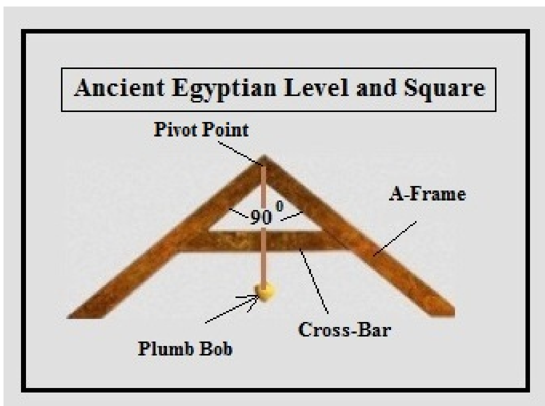
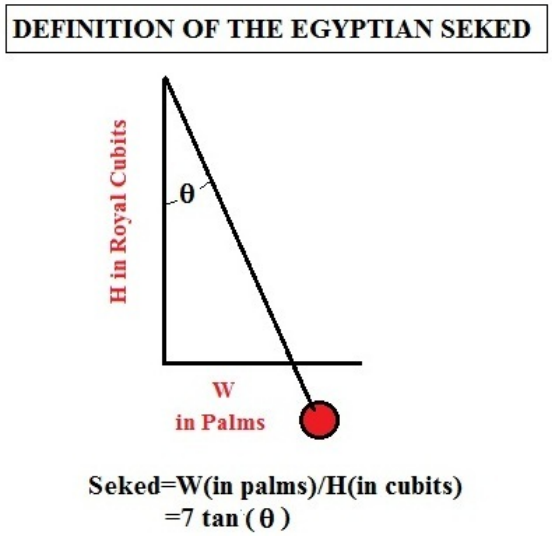
HOW DID THE ANCIENT EGYPTIANS MEASURE SLOPES?

About 4000 years ago the ancient Egyptians started building large pyramidal structures as burial places for their kings. The most impressive of these pyramids is the great pyramid of Cheops (Khufu) on the Giza plateau just outside of present day Cairo. It had an original height of H=146.59m (280 royal cubits) and a base side-length of L=230.35m(440 royal cubits). The cubit had a length of 52.5cm(20.7inches) and supposedly represents the distance from a man's elbow to the tip of his middle finger. For the construction of all Egyptian pyramids the cubit was the basic unit of length. A very important aspect of pyramid construction was the accurate determination of the slope of a surface relative to the horizontal. This was made possible via measurements using a plumb-bob level and a right angled square combined into a single instrument as shown-



It is our purpose here to show how this instrument was used to measure slopes in pyramid construction both at the stone quarries and during the actual placement of the stones at the pyramid site.

Our starting point for understanding the functioning of the Egyptian level-square, is to first discuss how the ancient Egyptians measured angles. Unlike in later centuries, they did not measure angles directly but rather used the concept of a slope which is equivalent to the tangent of an angle. Their angle related measure was the **seked**. It is defined as the ratio of two lengths of the sides of a right triangle taken in a certain way as shown in the following picture-



The measure is based upon the following equality-

1 royal seked=7 palms=28 digits

, where the seked is the length from a man's elbow to the tip of his middle finger, a palm the approximate width of the human hand, and the digit the width of a finger. So 4 digits=1 palm=1/7th seked. The ratio of the two shorter sides of the triangle shown are measured in cubits for the vertical side (H) and in palms along the horizontal side(W). Thus –

$$Seked = \frac{W \ in \ palms}{H \ in \ cubits} = 7\tan(\theta)$$

Here θ is the angle between the vertical and the plumb-bob line shown in the above figure. The 7 is due to the fact that one cubit equals seven palms. A table relating the seked to the tan(θ) and θ follows-

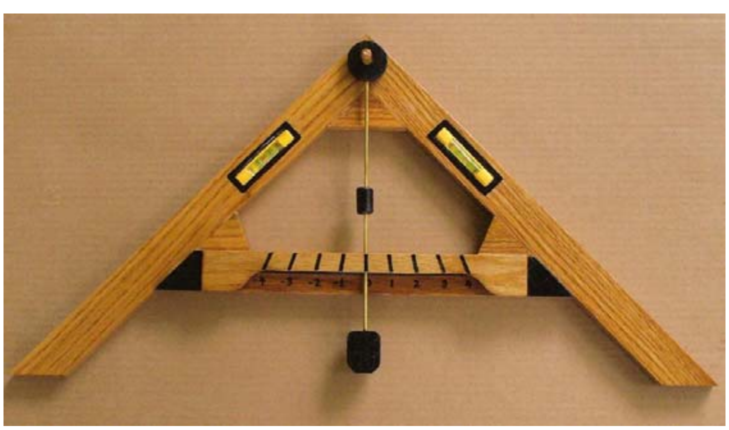
Seked	Tan(θ)	θ(in degrees)
1	0.1428	8.1268
2	0.2857	15.944
3	0.4285	25.873
4	0.5714	29.743
5	0.7142	35.534
6	0.8571	40.601
7	1	45
8	1.1428	48.814
9	1.2857	52.125

If we now take the Cheops pyramid where H=280 cubits and half the pyramid base is 220 cubits, we find the slope is-

$\tan(\theta)= 0.7857$ which means that $\theta=38.16^{\circ}$ relative to the vertical

Its compliment is $90^{\circ}-38.16^{\circ}=51.84^{\circ}$. This is precisely the angle the sides of the pyramid make with respect to the horizontal as indicated by the pre-carved angles present in the pyramid outer casing stones located along the bottom of the Cheops pyramid. Here the seked equals 220x7/280=5.5 which is equivalent to 5palms and two digits horizontal for each cubit rise vertical. An interesting point we wish to make(and apparently not recognized earlier by others) is that 220/3=73.33, 280/4=70, and $\mathrm{sqrt}(220^2+280^2)/5=71.22$.This looks suspiciously like a 3-4-5 Pythagorean triple, making one wonder if this fact was not something the ancient Egyptians wanted to incorporate into their pyramids. Certainly they knew that a right angle can always be constructed by having the sides of a triangle go as 3-4-5 .

Let us next go to the design of their angle measuring instruments. Clearly there was need to maintain a precise and constant inclination angle when building a pyramid from the ground up. Also the four slant edges of the pyramid must form 90 degree corners in the horizontal. These tasks can be accomplished with the combined level-square shown in the first figure above. The level function is accomplished by resting the two feet of the A-frame on a sloping surface and then noting at what point the string or rod on which the plumb-bob is attached crosses the central bar. By working the stone slope until a pre-selected seked is reached, guarantees the end stone slopes remain the same for all layers of the pyramid. The right angle can be achieved at both the stone quarry and at the construction site by cutting the stone surface in such a manner that it follows the right angle intersection dictated by laying the level-edge on top of the stone. It is also likely that once one precise cut stone had been prepared others could be constructed using wooden templates. This would be easier than repeating the level measurement for every end stone.
Recently we constructed a modernized version of the Egyptian level-square. A photo of the device is shown here-



We have used red oak for the arms and cross beam of the A-frame and attached a 1/8" diameter brass metal rod at a pivot point at the top of the level and placed a plumb-bob at the bottom of the rod. Instead of sekeds, we have marked the central bar in slope intervals of 1/10th. Thus for a 30 degree slope, where $\tan(\theta)=1/\text{sqrt}(3)$, the metal rod will cross the cross-bar at $10/\text{sqrt}(3)=5.77$ units from the neutral point. In terms of sekeds the answer would be $(7/10)5.77=4.03$. I have also imbedded two bubble levels along the arms to allow one to measure right angles between side walls and floor or ceiling of a standard rectangular structure when holding the level-square vertically. The slideable cylindrical plug along the metal rod above the cross bar allows one to immobilize the metal rod at a pre-selected slope and then adjust the footing.

A final interesting observation is that as late as the 18 hundreds AD carpenters world-wide were still using forms of the Egyptian square as a tool to allow precise right angle saw cuts.