

ENERGY DENSITY

Most of us back in our elementary physics courses in high school and undergraduate college learned about the various different forms of energy ranging from kinetic energy $E_k = (1/2)mv^2$, through chemical energy E_c , to nuclear where $E_n = mc^2$. We want here to look at the concept of energy content per volume and use it to point out some reasonable directions the world might take to meet its future energy needs.

Let us begin with kinetic energy and think of a 1kg of mass such as a wooden spear thrown at 20m/s. Its kinetic energy is-

$$E_k = (1/2)mv^2 = (1/2)1 \times (20)^2 = 200(\text{J})$$

Here J stands for joule the unit of energy in the MKS system, m is the mass expressed in kilograms, and v the speed in meters/second. Assuming the spear has a volume of $V = \pi R^2 L = 10^{-3} \text{ m}^3$, one comes up with an energy density of

$$D_k = (mv^2)/(2V) = 2 \times 10^5 (\text{J/m}^3)$$

The value of D_k is thus seen to be proportional to the product of the material density $\rho = m/V$ and the square of the speed. For a near earth satellite of mass density of 2000 kg/m^3 moving in orbit at $v = \sqrt{gR} = 7.9 \times 10^3 \text{ m/s}$, the kinetic energy density becomes $D_k = 6.24 \times 10^{10} \text{ J/m}^3$. This is some 300,000 times more than that of the moving spear. One can also talk about the kinetic energy density of a high speed rifle bullet made of lead (mass density $\rho = 11.34 \times 10^3 \text{ kg/m}^3$) moving at $v = 500 \text{ m/s}$. Its energy density is $D_k = 1.42 \times 10^9 \text{ J/m}^3$. Its energy density is thus 44 times less than the satellite but some 7100 times larger than the thrown spear.

The question now arises how are the kinetic energy densities of the bullet and the satellite so much larger than that of the thrown spear? The answer clearly is that the large kinetic energy densities have come about by conversion of chemical energy derived from the burning of rocket fuel and from the combustion of gun powder. It is also obvious from the bullet case that relatively small amounts of explosive powder contain a great deal of energy. Let us explore how much chemical energy is typically available in a combustion process. Take the case of burning in a controlled manner an oxygen-hydrogen stoichiometric gas mixture originally in a cryogenic liquid state. During combustion water is produced and 572kJ/mole of heat and kinetic energy are released. A kilogram of water contains 55.5moles, so that we can say that the energy density stored in the liquid oxygen-hydrogen mixture is-

$$D_c = 55.5 \times 572 \times 10^6 = 3.17 \times 10^{10} (\text{J/m}^3)$$

This number is quite high compared to most kinetic energy densities and shows why it takes relatively little volume of stored chemical energy in a bomb to produce very high velocity fragments. A typical roadside bomb in Afghanistan probably contains some 30kg of high explosive inside a metal casing of comparable mass. We can estimate that the

explosion of such a bomb will release about 10^8 J of energy which will produce a deadly spherical shock wave accompanied by flying casing fragments initially moving at supersonic speeds.

The chemical energy stored in gasoline is about 45 megajoules per kilogram and its density is about 720kg/m^3 . Thus its energy density is $D_c = 3.24 \times 10^{10} \text{ J/m}^3$ or about the same as that for a liquid oxygen-hydrogen mixture. It should be remembered that stoichiometric gaseous mixture of O_2 and H_2 will have an energy density about a thousand times smaller. A fact which should be kept in mind by individuals proposing the use of uncompressed gaseous hydrogen as a means of automobile propulsion.

Energy stored in batteries is quite small compared to that of gasoline. Typically a lead-acid battery stores about 146 kJ/kg , so that its energy density is –

$$D_c = 146,000 \times 5000 = 7.3 \times 10^8 (\text{J/m}^3)$$

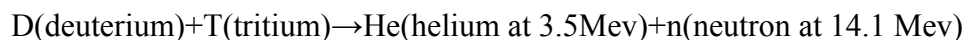
This means its energy density is some 44 times smaller than that for gasoline. Again one sees that batteries, although great for short trips in golf carts, are not able to effectively compete with gasoline as an energy source for longer distance transportation because of their need for frequent recharging.

Next we go to the energy density stored in nuclear material. In fission a uranium U235 is split by a neutron into two smaller fragments (say barium and krypton) plus the release of three neutrons capable of producing a chain reaction. In one of these fission processes about $200\text{Mev} = 3.2 \times 10^{-11} \text{ J}$ of energy is released in the form of kinetic energy and heat. This means that if one cubic meter of U235 containing about $1/(2.7 \times 10^{-29}) = 3.7 \times 10^{28}$ atoms were to completely fission, the energy released would be–

$$D_n = (3.2 \times 10^{-11})(3.7 \times 10^{28}) = 1.18 \times 10^{18} (\text{J/m}^3)$$

This is a truly large energy density exceeding the best chemical storage by some seven orders of magnitude. The mass converted to energy in this cubic meter of U235 would, according to Einstein's formula, be $m = E/c^2 = 1.18 \times 10^{18} / (3 \times 10^8)^2 = 13.1 \text{ kg}$ or about $13.1 / (19900) = 0.06\%$ of the original mass. In an actual atomic bomb the energy release per volume of fissioning uranium is less than these numbers by a couple orders of magnitude but still very large. The Hiroshima bomb "Little Boy" contained 64 kg of U235 but only about 1 kg of that amount actually underwent fissioning.

Next we come to thermonuclear fusion. Here one has two isotopes of hydrogen at very high temperature and pressure fusing into a helium atom plus the release of an energetic neutron. The easiest way to achieve fusion involves the reaction–



The deuterium can be separated from normal hydrogen but the tritium must be produced in a nuclear reactor and so will be quite expensive and has only a limited half-life. For the

reaction to occur at a reasonable rate the temperature will have to be about a billion of degrees Kelvin and thus presently can be achieved only within stars and hydrogen bombs. So far lab tests on fusion have failed to reach a break even point. A cubic meter of a frozen deuterium-tritium mixture will contain about 10^{30} atoms and so if brought up to fusion temperatures will release 17.6×10^{30} Mev of energy and have an energy density of-

$$D_n = 17.6 \times 10^{30} \times 1.6 \times 10^{-13} = 2.82 \times 10^{18} \text{ (J/m}^3\text{)}$$

This number is seen to be comparable in value to U235 and Pu239 fission reactions. However the advantage of hydrogen fusion is that there are no radioactive particles formed in the initial reaction before neutrons hit the containing walls of a thermonuclear reactor. Also there is no real volume restriction on the amount of D and T which can be used in a thermonuclear reaction. The largest hydrogen bomb ever exploded was the Russian 1961 “Tsar bomb”. It is estimated to have had a yield of 50 megaton equivalent of TNT which is some 3000 times larger than the yield of the 1945 “Little Boy” bomb dropped on Hiroshima.

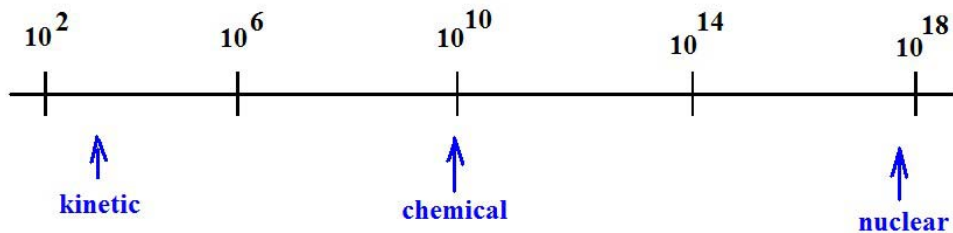
A remaining energy creation reaction observed at the atomic level, but unlikely to ever be put into practice on a larger scale (Star Trek excepted), involves the total annihilation of two masses such as between an electron and a positron yielding only high energy gamma rays obeying the $E=mc^2$ law. Here a fixed volume of matter and antimatter if it had a mass density of water ($\rho=1000\text{kg/m}^3$) would have an energy density of -

$$D_n = 1000 \times (3 \times 10^8)^2 = 9 \times 10^{19} \text{ (J/m}^3\text{)}$$

The number would become even higher when talking about neutron stars where the nuclei essentially touch and hence would have a density some 36 orders of magnitude higher than the huge value of D_n just given.

We can summarize all of the energy density discussions above in a single one line chart as follows-

ENERGY DENSITY IN JOULES PER CUBIC METER



What is very clear from this figure is that **we are increasing by seven to eight orders of magnitude the energy density as one goes from a typical kinetic to a chemical to a nuclear process.** Another way to look at this is to note that burning 1 m^3 of coal produces about the same energy as having $10,000 \text{ m}^3$ of water pass through a turbine at Hoover dam, to the energy produced by fissioning less than $10^{-6} \text{ m}^3 = 1 \text{ cc}$ of U235 in a nuclear reactor. This observation should be an important consideration when discussing the world's ever increasing energy needs for a population soon to exceed seven billion people. **It is unlikely that alternative energy sources such as wind and hydro(both kinetic sources) can ever approach in capacity our presently used fossil fuel sources(chemical) and in turn that chemical fuels will never be able to approach in capacity our almost certain increased use of nuclear power in the future.** It seems to me that people should realize this and concentrate on finding ways to solve the nuclear waste and potential meltdown problems instead of playing around with alternate energy sources such as solar and tidal power. In my view the best use of solar power will remain the growing of food. Also accelerated research on improved battery storage of electricity and the use of hydrogen for future transportation needs should be encouraged.