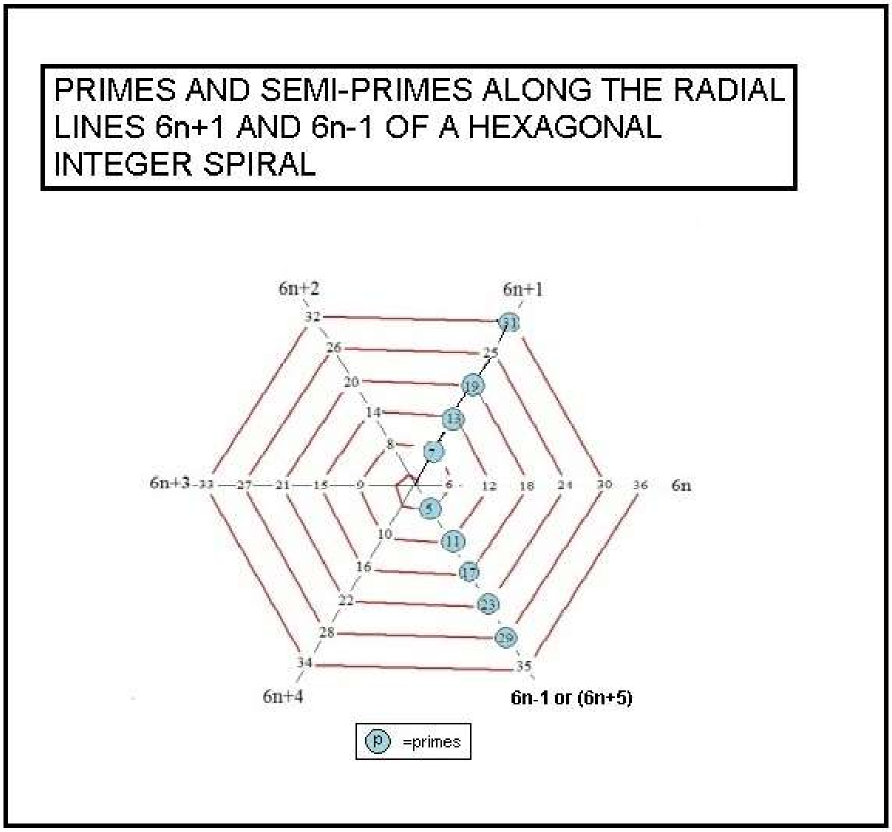
GRAPHICAL LOCATION OF THE TWO PRIME COMPONENTS OF ANY SEMI-PRIME

We have shown in earlier notes on this web page that all primes and semi-primes satisfy 6n$\pm$1, when integer n equals one or greater. This fact is well described by the following hexagonal integer spiral -



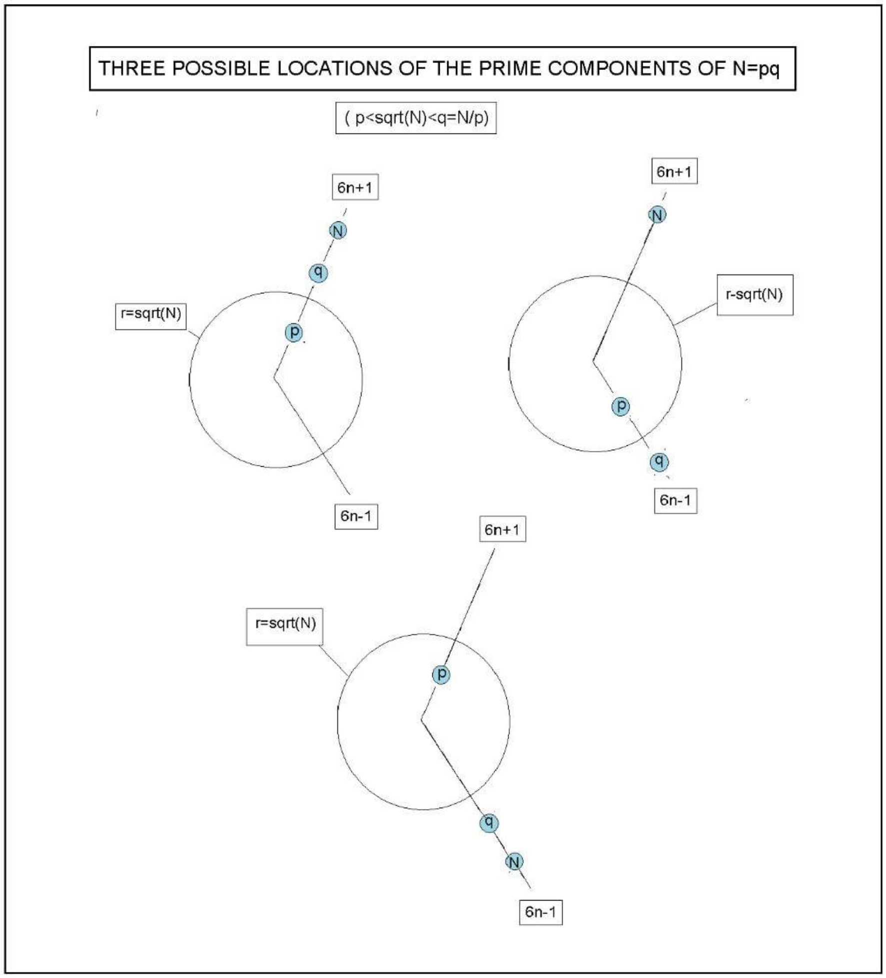
Here all primes lie along either 6n+1 or 6n-1. Also semi-primes N=pq, such as 25 and 35, lie along these same two radial lines. We wish in this article to locate the components for several specific semi-primes N=pq and present the results in geometrical form.

We begin with any general semi-prime-

$$N=pq=6s\pm1$$

Here s is an integer of size 1 or greater. This semi-prime lies along the plus or minus sixty degree radial line 6s+1 or 6s-1. The prime components p and q also lie along one of the two radial lines on opposite sides of a circle of radius r=sqrt(N).

There are three separate distinct configurations possible as shown in the following-



One finds the prime component p inside the circle of radius sqrt(N) and q outside the circle. Once either p or q are known the other follows from the N=pq definition. To see along which radial line N lies one needs to only perform a mod(6) operation. If N mod(6)=1, N lies along the 6n+1 line. If N mod(6)=5, N lies along the 6n-1 line.

Let us next look at the geometry of some specific Ns. Begin with-

N=24949 where N mod(6)=5 , s=4155, and sqrt(N)=157.89

So we know at once that N lies along the downward 6m-1 radial line with p

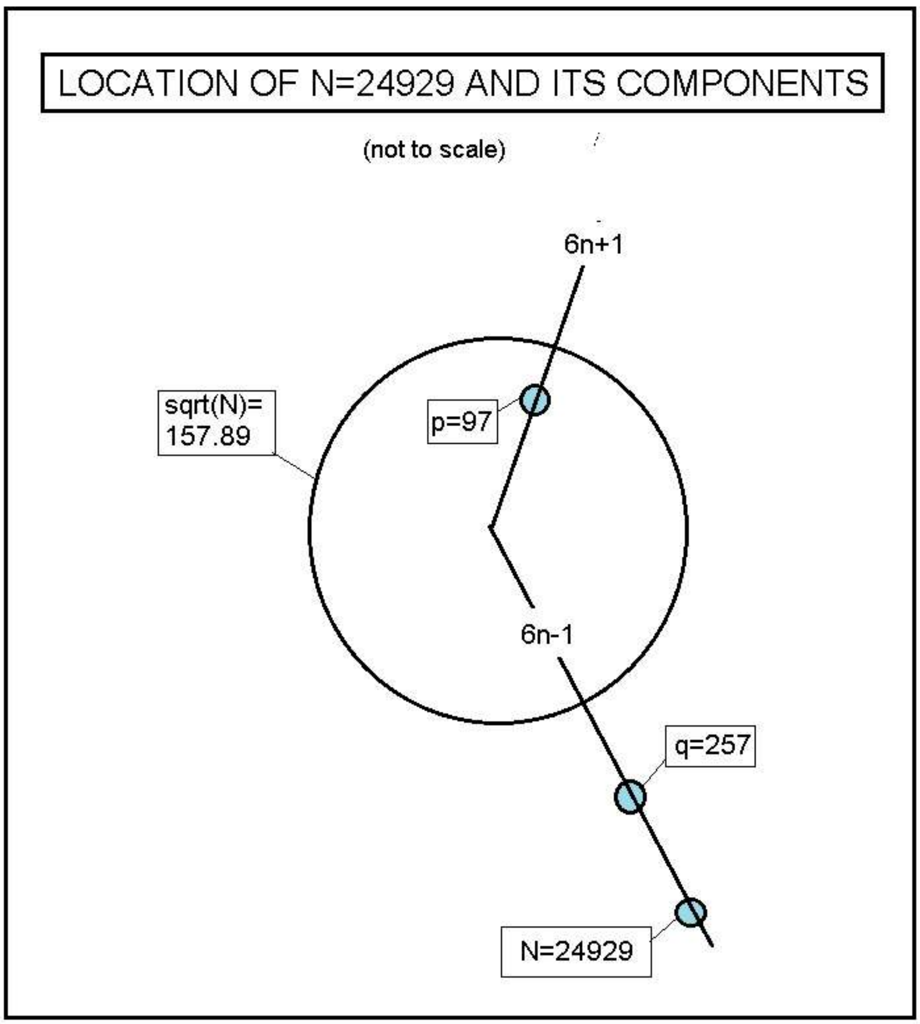
<158<q. From the above graph, p has the form 6n+1 and q of the form 6m-1.Multiplying things together yields-

$$(6n+1)(6m-1)=6(4155)-1$$

This is equivalent to-

$$m=[4155+n]/[6n+1]$$

It solves as n=16 and m=49, yielding the factors p=97 and q=257. We have the (not to scale) geometric picture-



As the second specific semi-prime consider-

N=42607 where N mod(6)=1 and sqrt(N)=206.41

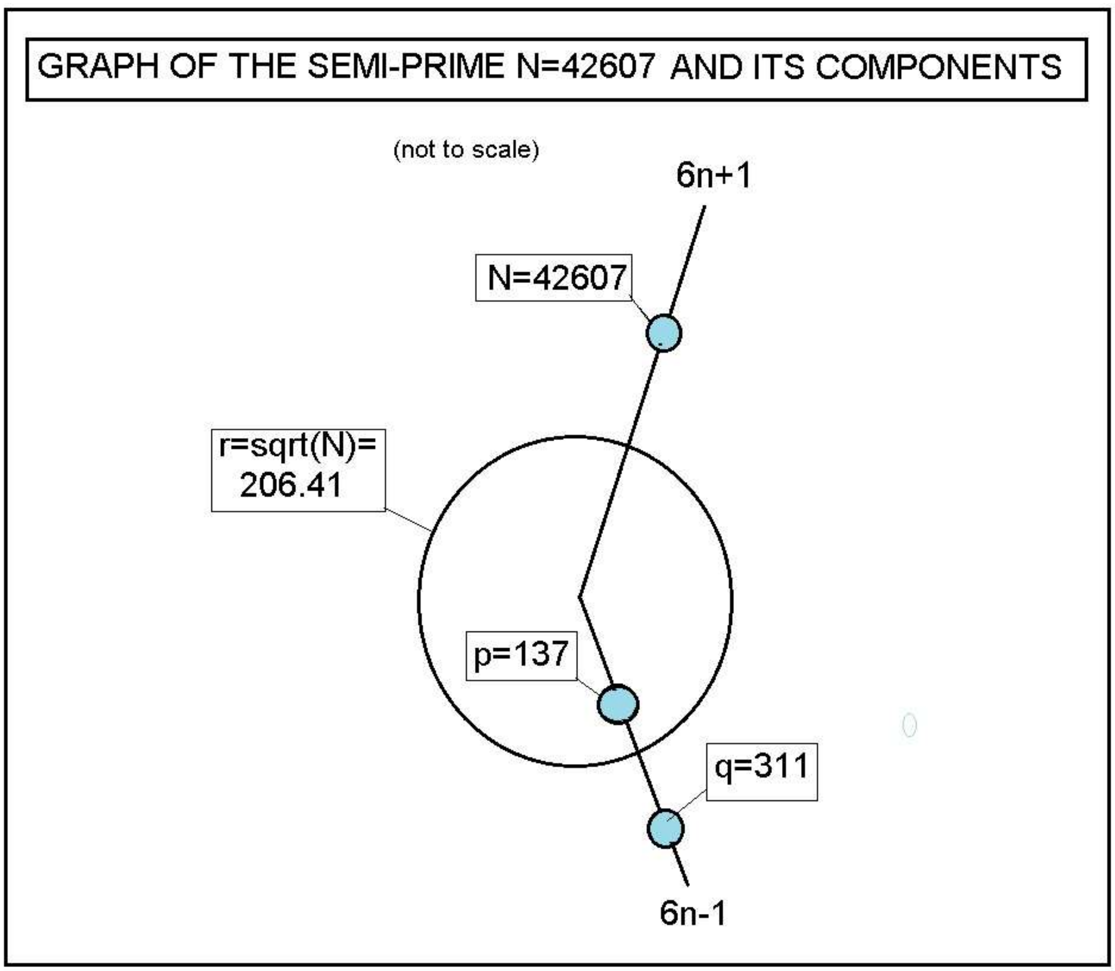
Here, according to the first generic circle graph above, there are two possible different locations for p and q. In one case we have p=6n+1 and q=6m+1 while in the other one has p=6n-1 and q=6m-1. The fastest way to see where p lies is to run the program-

N:=42697; for n from 0 to 60 do ({n,N/(6*n+1),N/6*n-1})od;

In a split second it yields the answers n=23 for q=311 and n=52 for p=137. So we have –

p=6(23)-1=137 and q=6(52)-1=311

So both p and q have mod(6)=5 meaning they lie along the 6n-1 radial line. Here is the picture-



We can also generate a picture where N, p , and q all fall on the line 6n+1. All one has to do is write down-

$$(6n+1)(6m+1)=6s+1$$

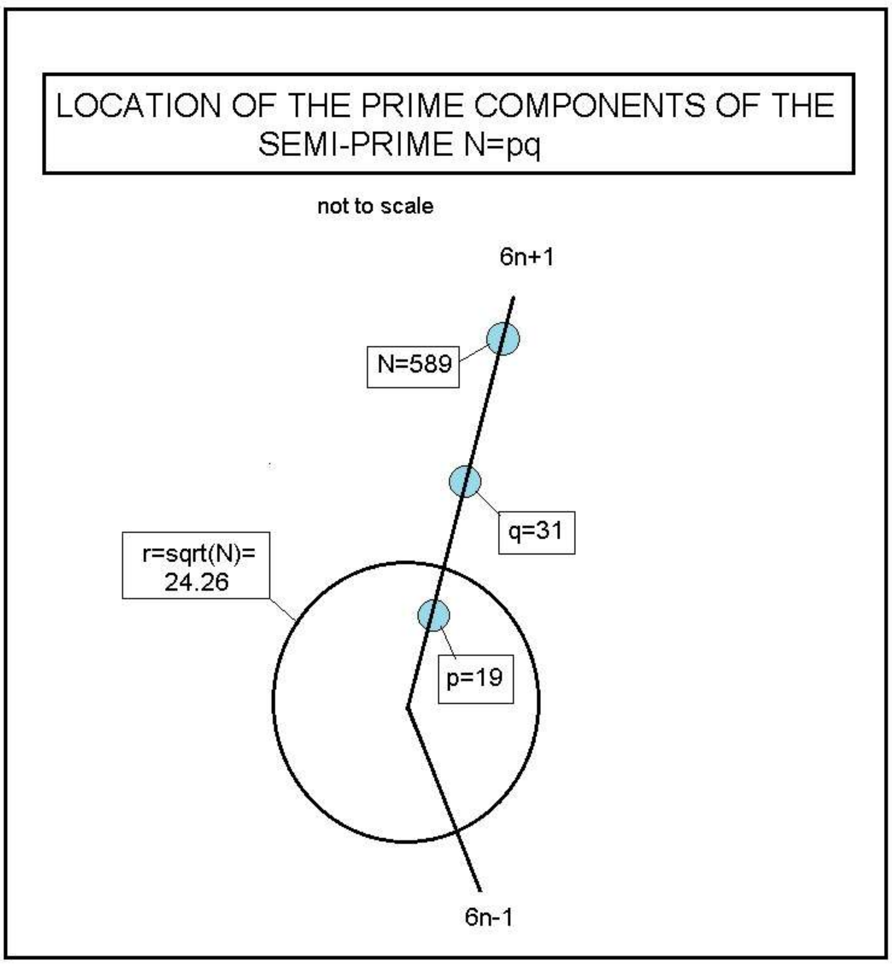
This is equivalent to-

$$6nm+(n+m)=s$$

Now taking n=3 and m=5, we get 90+8=98=s. So we get the numbers-

$$p=19 \quad , \quad q=31 \quad , \quad N=6(98)+1=589$$

The prime location geometry for this last N looks as follows-



Note that this reverse procedure for finding the p and q locations works only as long as p and q are both primes. N will always be composite.

We have shown that any semi-prime and its prime components can be projected geometrically to lie strictly along two radial lines without accept ion. A mod(6) operation on N starts the process. This is followed by an evaluation and location of p and q along one or both of two radial lines, as originally found by us several years ago while constructing hexagonal integer spirals. The p and q evaluation can be carried out by computer and several different methods are available for doing so including an earlier technique, not discussed here, involving the sigma function of number theory.

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