FRIEZES , DEFINITION AND CONSTRUCTION

A frieze as defined in mathematics is a 2D pattern repeated indefinitely in a given direction. Such patterns occur in architecture, wall paper edges , rug designs, and printing among many other applications. Two examples of friezes used by the ancient Greeks in temple design and the ancient Romans in floor mosaic construction follow-

EXAMPLES OF ANCIENT FRIEZES

Greek Meander

Roman Vitruvian Wave Floor Mosaic

Any frieze can be defined mathematically in terns of a basic 2D design g(x,y) confined to a rectangular region 0<x<a and 0<y<b which is then translated to the right repeatedly using Heaviside Function representations. It is the purpose of this article to show how such friezes are constructed.

We begin by looking at perhaps the simplest frieze which can be generated mathematically. It begins with defining a basic rectangle of length a=2 and height b=1. Within this rectangle we define a function-

$$g(x,y)=\left\{\frac{1\ \ for\ 0<x<1}{0\ \ for\ 1<x<2}\right\}$$

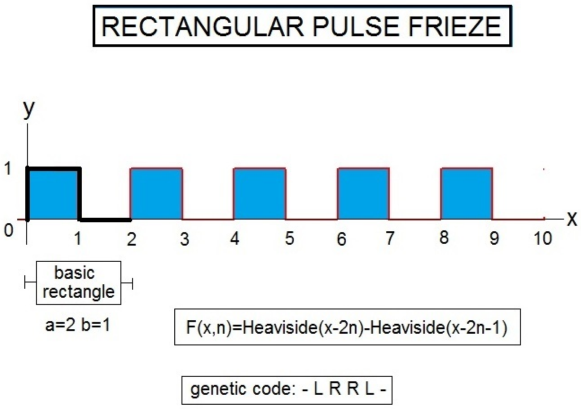
In terms of a Heaviside Step Function we can write-

g(x,y)=Heaviside(x-0)-Heaviside(x-1)   for   0<x<2

To generate the entire frieze we use the MAPLE command-

plot(sum((Heaviside(t-2*n)-Heaviside(t-2*n-1)), n=0..11),t=0..12,thickness=2,scaling=constrained, axes=none);

This produces, after adding a few additional features, the following graph-

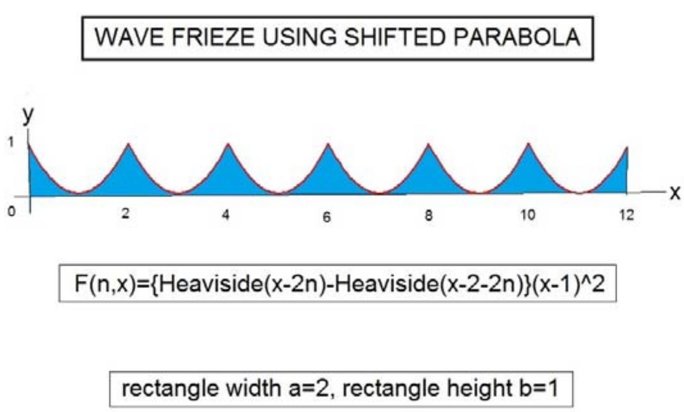


Notice all information present in this frieze is contained in the first rectangle. This fact allows us to describe these multiple pulses by the genetic code L R R L.
That is, first go one unit to the left, follow this with one unit to the right followed by another unit to the right and ending with one unit to the left.

Consider next a parabola placed into a 2x1 rectangle . The equation for this parabola in 0<x<2 is given by-

(Heaviside(x-0)-Heaviside(x-2))*(x-1)^2

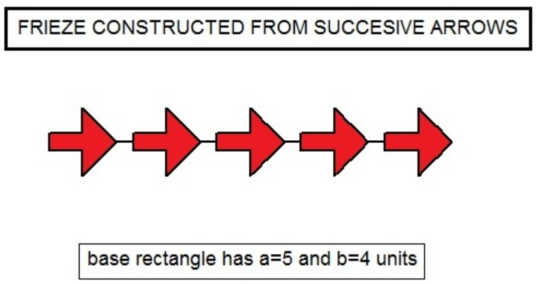
To get the entire frieze we introduce the substitution x->x-2n and sum over the integer values of n. This produces the following wave frieze-



More elaborate friezes may be constructed using combinations of reflection and t translation of the base rectangle, For example, a repetitive arrow frieze with a=5 and b=4 is constructed as follows-

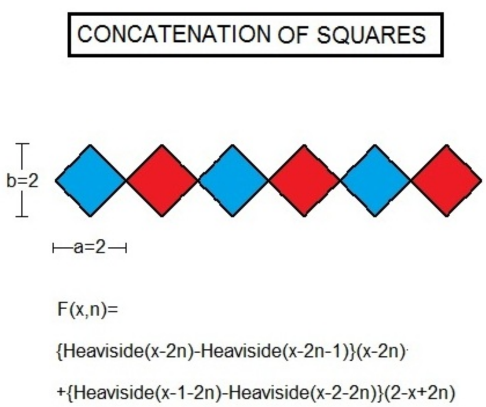
K:=(1/2)*(Heaviside(x-0-5*n)-Heaviside(x-2-5*n))+(4-x+5*n)*(Heaviside(x-2-5*n)-Heaviside(x-4-5*n));
L:=sum(K,n=0..15):
plot({L,-L},x=-0.001..24,color=black,thickness=2,scaling=constrained,axes=none);

The resultant graph looks like this-



We see from the computer program that translation in the x direction equals 5 units. Also a reflection about the x axis yields the bottom parts of the arrows.

Another frieze which can be constructed using the above three-line program is for a concatenation of squares. Here is the result-



One could go on using additional mathematical formulations including ones which allow both x and y variations within the base rectangle. Also one could extend friezes to simultaneous repetitions in both the x and y directions forming for instance the hexagonal patterns associated with honey combs. I leave such suggestions for a later article, but I do want to finish things up with an alternative and less mathematical way to generate friezes for any elaborate design within the base rectangle. The approach is to copy a given pattern in the base rectangle and transfer it to a program such as microsoft paint and then produce n copies and hook them together into an x-axes array. Let me demonstrate the result for a frieze based on a single base rectangle containing a jpg of a Roman floor mosaic-

FRIEZE CONSTRUCTED FROM A SINGLE ROMAN FLOOR MOSAIC USING MICROSOFT PAINT

employing the COPY and PASTE commands

I first saw one of these 2000 year old floor mosaics at Ostia near Rome during my Fulbright year abroad in 1961-62. The skill these ancient artists had for constructing intricate patterns using small pieces of different colored stone is truly amazing.

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