GOLDBACH PAIRS AND GOLDBACH'S CONJECTURE

Back in 1742 Christian Goldbach(1690-1764) wrote in a letter to the great Swiss mathematician Leonard Euler that-

ANY EVEN NUMBER (FOUR OR HIGHER) CAN BE EXPRESSED AS THE SUM OF TWO PRIMES

That this statement is correct for all smaller even numbers is shown in the following table-

4=2+2
6=3+3
8=3+5
10=3+7 or 5+5
12=5+7
14=3+11 or 7+7
16=3+13 or 5+11
18=5+13 or 7+11
20=3+17or 7+13
22=3+19, 5+17, or 11+11

Making such a list undoubtedly was the route Goldbach took to arrive at his conjecture. However, a general proof for all even numbers greater than four eluded Euler and others after him so that the the Goldbach Conjecture remains one of the most important remaining unproven theorems of number theory although numerically it has been verified to be correct for even numbers as high as $N=10^{18}$. It is our purpose here to attempt to give a proof of Goldbach's Conjecture by making use of our newly discovered fact that all primes five or greater must have the form $6n\pm1$.

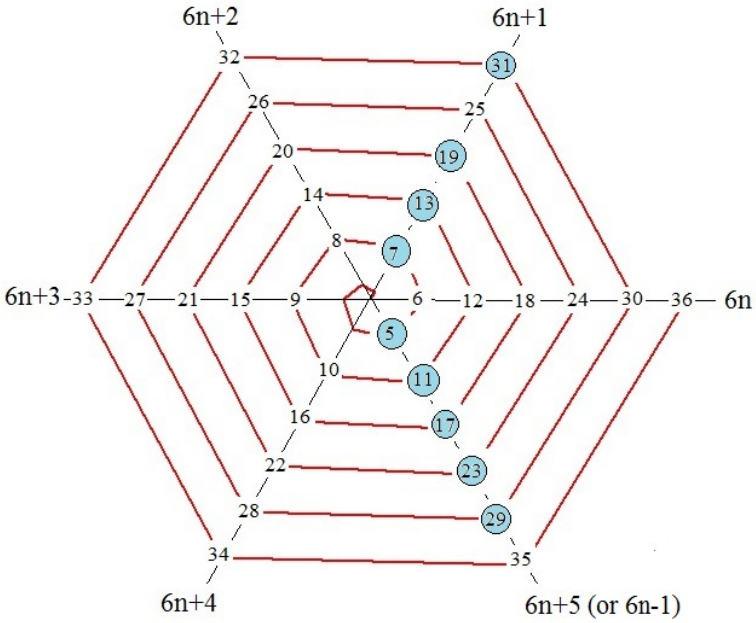
We begin by noting in the above list that larger even numbers can be represented by multiple prime pairs (p_n,p_m). We call such prime number combinations Goldbach Pairs. Thus the even number N=22 can be represented by three Goldbach pairs (3,19), (5,17), and (11,11). What we note is that as N increases the larger the number of Goldbach Pairs become, going from 1 pair ar N=4 to 150 pair at N=4800. If one can show in general that all larger even Ns always consist of multiple Goldbach pairs, then one pair can always be chosen from the list to verify Goldbach's Conjecture.

To find the number of possible multiple pairs for a given even N we make use of our earlier discovered observation that-

ALL PRIME NUMBERS GREATER OR EQUAL TO FIVE MUST HAVE THE FORM 6n±1

This is a necessary but not sufficient condition since there are also composites of this form. In several earlier notes we have introduced a graphical way to plot all primes five or greater as points along two radial lines 6n+1 and 6n-1 crossing a hexagonal integerr spiral as shown-

HEXAGONAL INTEGER SPIRAL AND THE LOCATION OF THE FIRST FEW PRIMES FIVE OR GREATER



Unlike an Ulam Spiral , this representation aligns the primes along just two radial lines. Now all even numbers N in this diagram lie along the radial lines 6n, 6n+2 and 6n+4. The value of N mod(6)=0, 2, or 4 determines along which of these three lines N lies. We observe that-

(1) for N mod(6)=0 we have N=(6n+1)+(6m-1)=6(n+m).

(2) for N mod(6)=2 we get N=6n+1)+(6m+1)=6(n+m)+2

(3) for N mod(6)=4 we have N=(6n-1)+(6m-1)=6(n+m)-2

Let us now proceed in calculating the Goldbach Prime Pairs. It will be sufficient to just look at the case where N=6(n+m)=6k. This means those even numbers N=6k=6,12,18,…

We use our PC to calculate these pairs starting with N=12. The one line computer program used will be-

for n from 1 to N/6 do {n,6n+1,N-1-6n,isprime(6n+1),isprimke(N-1-6n)}od

Once these numbers are found one eliminates those terms were one or the other prime pairs reads false, leaving us with the appropriate Goldbach Pairs for any number of the form N=6k. Let me demonstrate things for N=90=6(15). Here we get-

NINE GOLDBACH PAIRS FOR N=90=6(15)

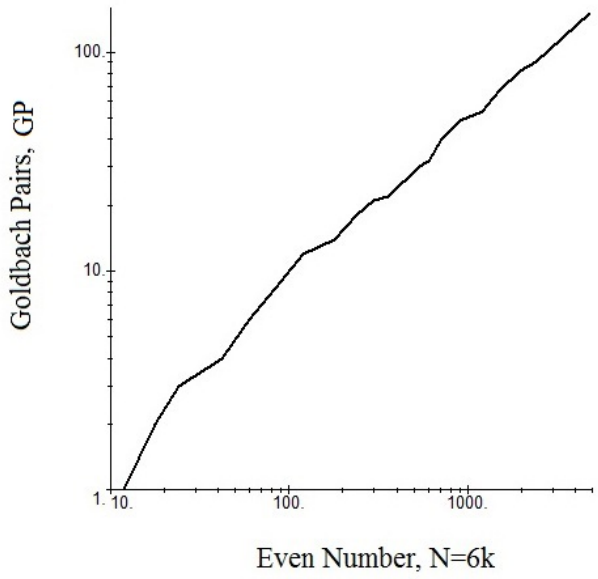
$N := 90$
$\{1, 7, 83, true\}$
$\{3, 19, 71, true\}$
$\{5, 31, 59, true\}$
$\{6, 37, 53, true\}$
$\{7, 43, 47, true\}$
$\{10, 29, 61, true\}$
$\{11, 23, 67, true\}$
$\{12, 17, 73, true\}$
$\{11, 13, 79, true\}$
$k := 15$
$GP := 9$

So that there are nine Goldbach Pairs (GP=9) for the even number N=90. We have calculated N =6k versus GP number at a total of twenty points getting the results-

{N.GP}= { [12,1], [18,2], [24,3], [42,4], [60,6], [90,9], [120,12], [180,14],[240,18], [300,21],[360,22], [540,30], [600.32], [720,40],[900,49], [1200,54], [1500,67], [1980,82],[2400,90], [4800,150] }

A logarithmic plot of these results follows-

GOLDBACH PAIRS FOR NUMBERS N=6k OVER THE RANGE 12<N<4800



We have here a non-smooth curve which for large N =6k trends approximately as-

$$GP=0.3289\ N^{0.7222}$$

Although we have not carried out calculations in detail for the other two groups of even numbers N=6k+2 and 6k+4, the trend in Goldbach Pairs is quite similar. For example for N=6(20)+2=122 and N=6(30)+4=184 the number of Goldbach Pairs equal GP=7 and GP=14, respectively.

What is quite clear from these results is that GP increases as a power of N meaning that for N≥12, for which the N=6n±1 forms apply, will have at least one Goldbach Pair available to confirm Goldbach's Conjecture. When this fact is taken into account together with the list given at the beginning of this article, it is clear that the Goldbach Conjecture is true for all even numbers N .

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