FINDING TWIN PRIMES

If one looks at a list of all positive integers 5 through 50 one finds the following pattern-

S(N)={5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50}

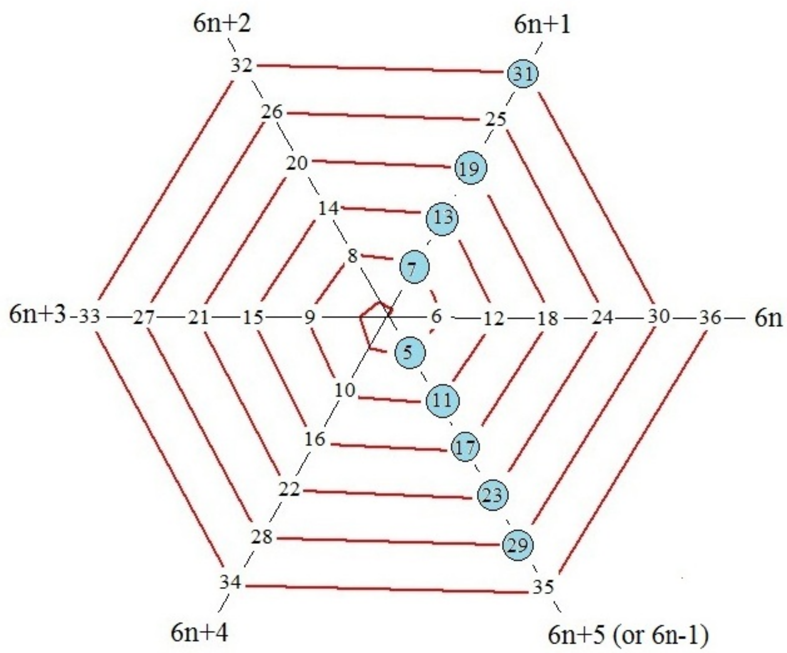
The black numbers constitute the primes which are characterized by being divisible only by themselves and one. The red numbers are the composite numbers characterized by being divisible by three or more integers. Thus, for example N=3467 has divisors (1,3467) and is a prime while N=6253 has divisors (1,13,17,169,481,6253) and is a compound number.

What we noticed several years ago is that all primes five or greater have the form N=6n±1 without exception although there can also be some compound numbers such as 25 which also have this form. To distinguish whether a number of the form 6n±1 is a prime or not you can use a simple prime test f= [σ(N)-N-1]/N =0 for primes and greater than zero for compound numbers. Thus N=25 yields f(25)=1/5 so 25 must be a compound number. We have termed f(N) the number fraction since it produces a unique integer fraction for any N. f(234567)=110192/234567 is a composite.

A very convenient way to represent all positive integers is by an integer spiral defined in polar form as-

$$[r,\theta]=[N,\exp(i\pi N/3)]$$

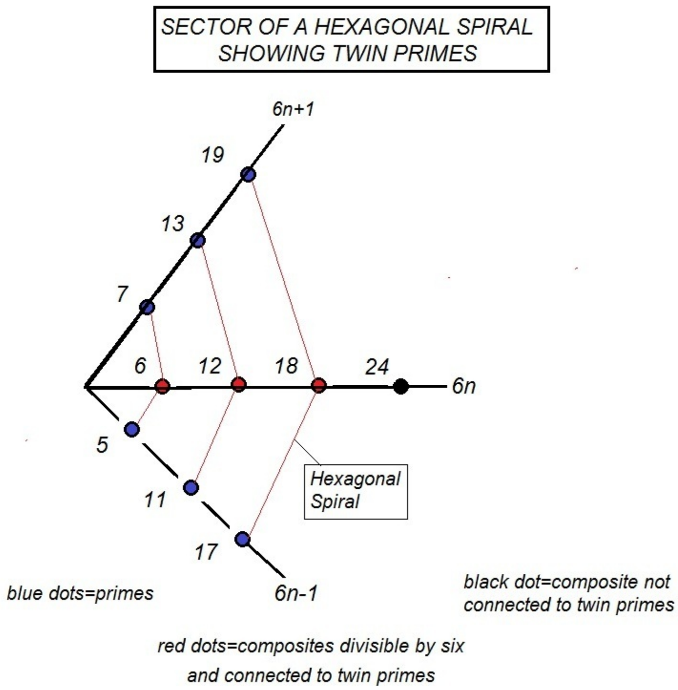
We can connect these points by straight lines to generate a hexagonal spiral were all integers will be found at the intersection of the spiral's corners and one of the six radial lines 6n, 6n+1, 6n+2, 6n+3, 6n+4, 6n+5. The hexagonal integer spiral which emerges looks as follows-



What is most interesting about this picture is that all prime numbers five or greater are found along only the 6n+1 and 6n-1 radial lines. Mathematicians have for years tried to make sense out of the appearance of the partial regularity of primes in the familiar Ulam Spiral without ever realizing the existence of the much simpler arrangement for primes shown above. (Shades of the Copernican versus the Ptolemian view of the solar system)

You will notice the primes marked in blue in the above graph often differ from each other by a change of just two digits in their numerical value. Such pairs are known as twin primes. Examples include [5,7], [11,13] ,[17,19],[29,31], etc. It is our purpose here to further discuss the properties of such twin primes making full use of the hexagonal integer spiral.

The first thing we do is to look at only the sector of the hexagonal spiral in the angle range $-\pi/3 \leq \theta \leq \pi/3$. This produces the following magnified picture-



Labeling the primes along 6n+1 as p_n primes and those along 6n-1 as q_n primes, we see at once that-

$$(p_n+q_n)/2=6n \quad \text{and} \quad p_n-q_n=2$$

Here N=6n is an even compound number lying along the 6n radial line. Thus we can state ,when both p_n and q_n are greater than three, that-

Twin Primes will only exist if n mod(6)=0 and both p_n and q_n are primes

Two examples of twin primes would be [59,61] and [281,283]. For the first case n=10 while in the second it is n=47. A non-twin prime occurs for n=155 where 6n=930 and although 929 is a prime, 931=7x7x19 is a composite.

One can easily write a one line computer program which picks out all twin primes in a given range of n, For the first few twin primes it reads-

for n from 1 to 40 do {n,6*n,isprime(6*n+1),isprime(6*n-1)}od;

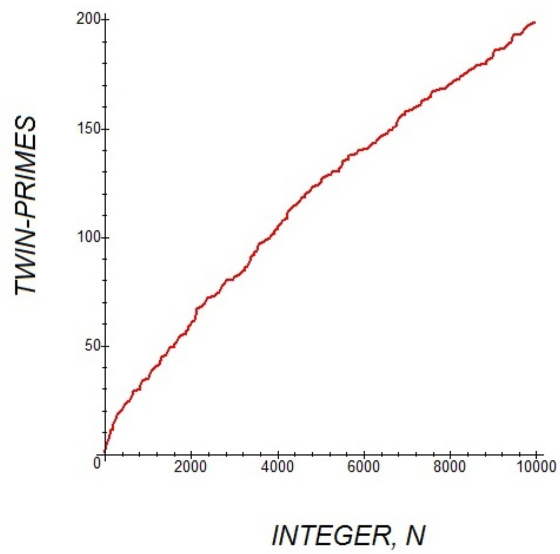
The computer output is-

{1, 6, *true*}	{18, 108, *true*}
{2, 12, *true*}	{23, 138, *true*}
{3, 18, *true*}	{25, 150, *true*}
{5, 30, *true*}	{30, *true*, 180}
{7, 42, *true*}	{32, *true*, 192}
{10, 60, *true*}	{33, *true*, 198}
{12, 72, *true*}	{38, *true*, 228}
{17, 102, *true*}	{40, *true*, 240}

So there are a total of 16 double primes lying between n=1 and n=40. That is between N=6 and N=240. The twin prime at N=240 is [239,241]. Note the twin prime of [3,5] has not been counted since it not covered by the 6n±1 restriction.

I have made a detailed count of the cumulative value C for twin primes as a function of N up to N=9930. The results are summarized in the following graph-

TWIN-PRIME COUNT FOR N from 6 to 9930



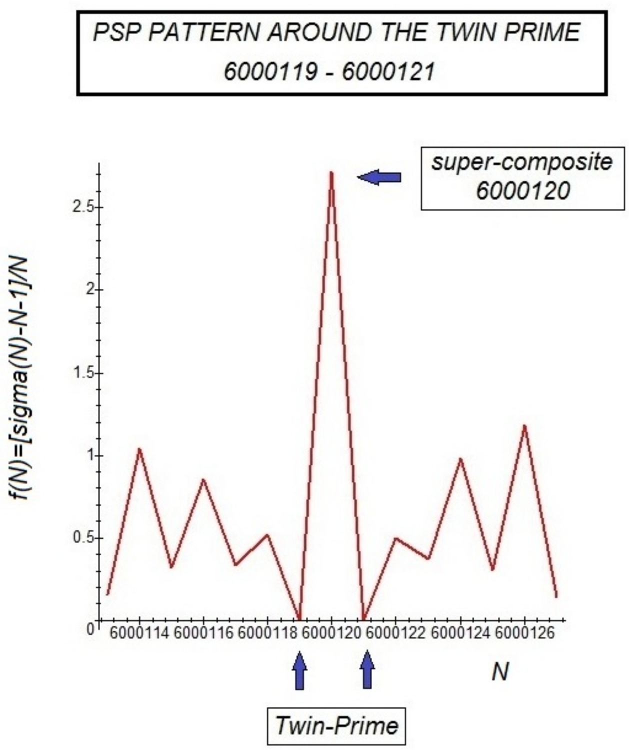
Twin-Prime at N=9930 ± 1 is the 200th if [3,5] is included

Notice that the twin-prime density decreases with increasing N but keeps climbing. This gives strong support for the existence of an infinite number of semi-primes. We note that if the case of [3,5] is included then the twin prime 9930±1 is the 200th. To generate these twin-primes we used a two step process. First we asked our PC to carry out the command-

for N from 1 to 9930 do {N,isprime(N+1), isprime(N-1)}od

Next we looked at batches of Ns of length 100 each and picked out those where both isprime(N+1), and isprime(N-1) were answered in the affirmative. The process took a couple hours of effort.

A final observation concerning twin-primes is that they sandwich in super-composites of the form N=6n. In terms of the number fraction f(N)={sigma(N)-N-1)}/N one gets interesting patterns since f(N±1)=0 while f(N)>1. Here is a typical graph showing f(N) in the neighborhood of a twin prime-



We see there the twin primes at N±1, for which f(N±1)vanishes, sandwiching the super-composite number N. We call this the PSP pattern standing for prime-super-composite-prime . Note the near symmetry of the pattern over the range N±6.

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