MANIPULATING WITH LOGARITHMS

It is well known that any number N can be expressed as a base b taken to a unique power x. This power is known as the logarithm of N with respect to the base b and is written as

$$\log_b(N)$$

Although the base can be any number, the most common ones are the bases b=10 and b=e=2.71828… . One calls $\log_{10}(N)$ the Briggsian or common logarithm and $\log_e(N)$ the natural logarithm. The common and natural logarithms are also written as log(N) and ln(N), respectively. This is the way they appear on hand calculators. As examples, we have log(86)=1.93449845…and ln(10)=2.302585093…

What makes these logarithms useful is that they can be used to multiply and divide large numbers using only simple addition and subtraction when employed in conjunction with a good table of logarithms. That is, we can write-

$$N \times M=b^\wedge[\log_b(N)+\log_b(M)] \quad \text{and} \quad N/M=b^\wedge[\log_b(N)-\log_b(M)]$$

Consider, as an example, the product of two numbers N=462 and M=683 with the Briggsian base b=10. The product reads-

$$462x683=10^\wedge[\log(462)+\log(683)]=10^\wedge\{5.499062679\}=315546$$

, with the number in the curly bracket for base 10 being the logarithm of the desired answer. To get the final answer one uses a logarithmic table. This way of multiplying and dividing with large numbers goes back to the early sixteen hundreds and is associated with the names of Joost Burgi, John Napier, and Henry Briggs. The method remained active through the end of WWII however became obsolete with the advent of hand held electronic calculators in the 1970s. When I went to high school in the early 1950s we were still taught the method. We point out that logarithmic calculations can be used with any chosen base b. To convert Briggsian logarithms to natural logs one simply uses the identity-

$$\ln(10)\cdot\log(N)=\ln(N)$$

This says if you have a logarithm table in either natural or common form the two differ by ln(10) for any number. Thus –

$$\ln(2)=\log(2)\cdot\ln(10)=0.301029995\cdot2.302585093=0.69314718\ldots$$

Let us carry out the evaluation of the following number using a logarithmic approach with base b=e.

$$N=(517 \times 31^\wedge3)/(\text{sqrt}(726 \times 1908)$$

$$= e^\wedge[\ln(517)+3\ln(31)-0.5\ln(726)-\ln(1908)]$$

$$=e^{[6.248042875+10.30196161-3.293775007-7.553810852]}$$

$$=e^{\{5.702418628\}}=299.5911246$$

This is the correct answer but it takes much longer to get than the split second result using my Casio hand calculator. The calculator produces-

$$N=299.59162$$

This last result again shows why calculations using logarithms have become obsolete. The main positive effect of having learned logarithmic calculations is that it made students in the good old days more aware of manipulations with different number bases and how to operate with them.

There still are a few areas where knowledge of logarithms is useful. This is especially true when dealing with equations containing $\log_b(x)$. Let us consider some of these equations and show their solutions.

Start with

$$2^x=5$$

and take the Briggsian log of both sides of the equation. This produces the solution

$$x=\log(5)/\log(2)=2.321928\ldots$$

A hand calculator confirms that $2^{2.321928}=5$. Take next the equation-

$$2^x+3^x=4$$

Applying the common logarithm to both sides produces-

$$x[\log(2)+\log(3)]=\log(4)$$

Hence we have-

$$x=\log(4)/\log(6)=0.77370\ldots$$

As a third example of a logarithmic equation consider-

$$\ln(x)+\ln[(x+1)^2]=3$$

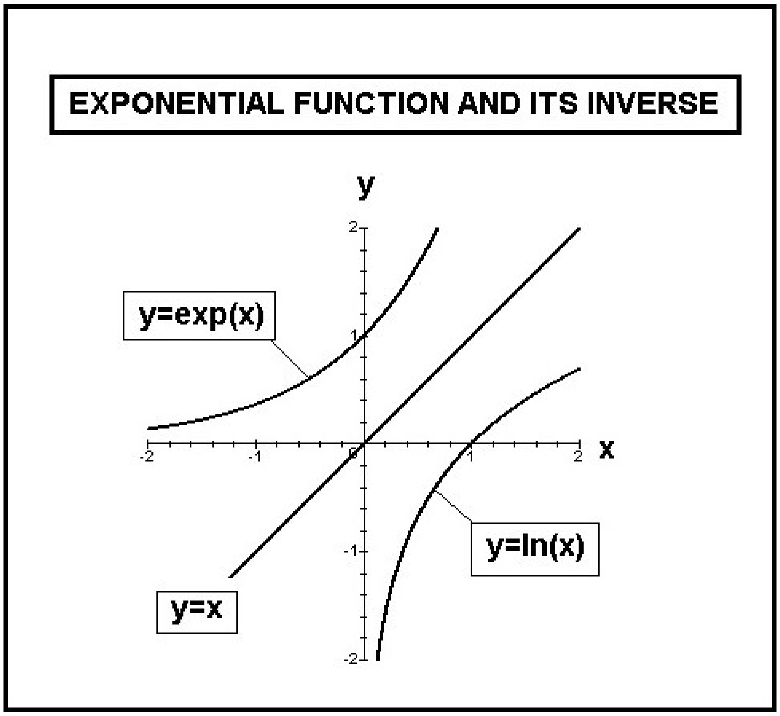
Combining the logs and then taking the exponent we find-

$$x(1+x)=\exp(3/2)$$

On solving this quadratic, we get the final result-

$$x=1.675244..$$

Note that there is also a second solution which is negative. Logs for such negative numbers do not exist as shown in the following graph-



This graph also shows very nicely how exp(x) and ln(x) are inverse functions of each other by replacing x by y.

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