FINDING THE NEAREST PRIME TO ANY LARGE INTEGER

About a decade ago we came up with a new way to represent all positive integers as points at the intersection of a hexagonal integer spiral and six radial lines emanating from the origin. These observations are summarized by the following graph-



By moving counterclockwise along the spiral one goes from 1 through N with six integers contained in each turn of the spiral. What is most interesting about this result is that prime numbers P greater than three all strictly lie along just the two radial lines $6n\pm1$ corresponding to P mod(6)=1 or P mod(6)=5. Since there are also some composite numbers such as 25, 35, and 77 satisfying the $6n\pm1$ criterion, we are able to make the somewhat limited statement that-

A necessary but not sufficient condition for a number greater than three to be a prime is that it has the form 6n±1

We have found no exception to this rule testing things out to numbers up100 digit length. For example the prime-

N=1155091181188912471405826728926946015236727784861

can be written as

={(192515196864818745234304454821157669206121297477) x 6}-1

This is equivalent to saying N mod(6)=5, so that N must lie along the radial line 6n-1 (or equivalent 6n+5) in the fourth quadrant. It is the purpose of the present note to show how one can quickly find the nearest prime P to any integer N.

We start my noting that N-N mod(6)=M lies along the 6n radial line in the above graph. So to get a prime we can write-

P=N-N mod(6)+6n ± 1 with n=0, $\pm 1,\pm 2$, etc

The nearest prime will be one of these solutions at small n. Let us demonstrate things by finding the nearest prime to N=55440. Here we have N mod(6)=0 so that-

P=M+6n±1=55440+6n±1

On carrying out the search –

for n from -5 to 5 do{55440+6*n±1, isprime(55440+6*n±1)}od;

we get the following primes-



Clearly there are two closest primes found at 55441 and 55439 both separated by one unit from N. Note that 55441 mod(6)=1 and 55439 mod(6)=5.

An alternate way to find the nearest prime is a graphical approach using the <u>number</u> <u>fraction-</u>

$$f(x) = \frac{\sigma(x) - x - 1}{x}$$
 x=0,1,2,3,...

Here $\sigma(x)$ is the divisor point function encountered in number theory and f(x) is a point function found earlier by us. You will note that f(x) = 0 for primes and f(x)>0 for composite numbers. So to find a prime number near integer N one simply needs to graph-

over the range -b<n

b and then collect those points where f=0. For N=55440 we get the following graph-



So we have (as before) the two nearest primes of 55439 and 55441 lying along the radial lines 6n-1 and 6n+1, respectively. When two primes are separated from each other by two units (as they are here) they are referred to as twin primes. Note that this graphical approach gives somewhat more insight into the problem than the earlier search method.

Take next a more complicated example corresponding to the 20 digit long number -

Here we find N mod(6)=1, so that M=N-N mod(6) lies along the 6n radial line. For such larger Ns it is generally more difficult to find the value of the number fraction f, so it is recommended that one instead resort to the search program shown in red above. For the present value of N we have the modified program-

for n from -3 to 3 do{M+6*n±1, isprime(M+6*n±1),N-(M±1+6*n)}od;

Keeping only those numbers which are prime we obtain the following-

PRIMES NEAR N= 84209175623541789235	
6n+1	6n-1
{42, true, 84209175623541789193}	$\{26, true, 84209175623541789209\}$
{36, true, 84209175623541789199}	$\{-46, true, 84209175623541789281\}$
{-18, true, 84209175623541789253}	
nearest prime	

P=N+18= 84209175623541789253

The smallest distance from N to the nearest prime is seen to be P=N+18.

As a third example we take the 69 digit long number-

N= 470246729145067566481388362634804346740547308753087957946131975269918;

Running a search program for -50<n<50 we find the nearest prime to be-

 $P{=}470246729145067566481388362634804346740547308753087957946131975269981$

Here we have a difference of N-(M+1+6(11))=-63. Also $P \mod(6)=1$ meaning this prime lies along the radial line in the first quadrant.

We have shown that one can find the nearest prime P to any number N. The procedure involves first placing N-Nmod(6) =M onto the 6n axes and then adding $6n\pm 1$ to it until the smallest n is found which makes the result a prime number. There appears to be no restriction on the size of the starting integer N.

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