A NECESSARY AND SUFFICIENT CONDION FOR A NUMBER TO BE PRIME

A prime number is defined as a number divisible only by one and itself. The fifteen lowest primes are-

$$P= \{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47\}$$

Regrouping these yield-

$$\{2,3\},\{5,11,17,23,29,\ \ ,41,47\},\{7,13,19,\ \ .31,37,43\}$$

So after neglecting two and three, we are left with two sequences 6n-1 and 6n+1given as-

$$6n-1=\{5,11,17,23,29,41,47,..\}$$

and-

$$6n+1=\{7,13,19,31,37,43,.\}$$

In these sequences we have dropped any composite numbers of this form. Note that the numbers in both sequences always differ by multiples of six. In modular arithmetic language we have –

$$(6n-1)\mathrm{mod}(6)=5 \qquad \text{and} \qquad (6n+1)\mathrm{mod}(6)=1$$

So it is clear that any prime above N=3 must have the form $6n\pm1$. But since some $6n\pm1$ numbers such as 25 and 35, remain composite, additional information is needed to *guarantee* primeness. This additional information is gotten by looking at the sigma summation function of number theory. It is defined for a prime number as-

$$\sigma(N)=!+N$$

which shows that the sigma function and N differ by just one. Hence we can conclude that-

Necessary and sufficient condition that a number N , five or greater, is prime requires that it have the form 6n$\pm$1 plus satisfy $\sigma(6n \pm 1) - (6n \pm 1) = 1$

Let us apply this law to several different numbers. Starting with –

N=3762935023

It has N mod(6)=1 but fails with $\sigma(N) - N$=390988625. So it is composite'

Next take-

N=57210093723349597

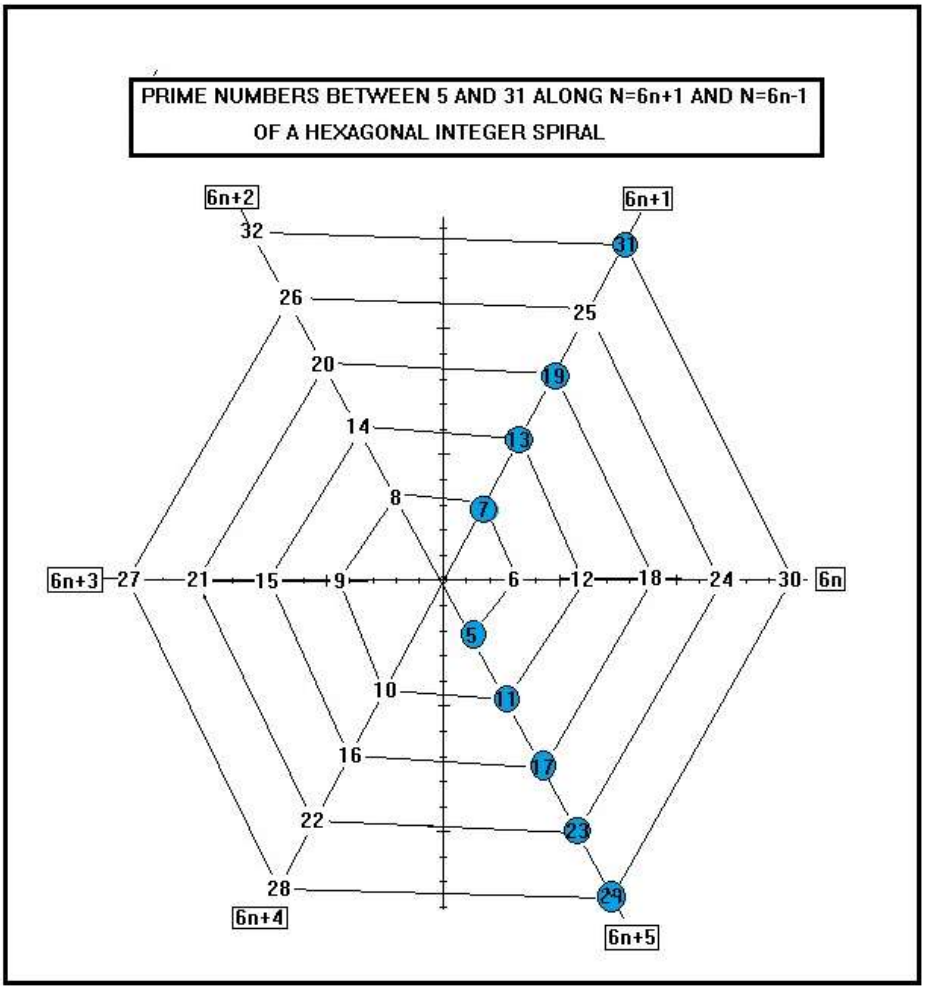
It has N mod(6)=1 and σ(N)-N=1. Hence this last number is a prime.

As the last number take the thirty digit long--

N=587230018356723401290557623361

It must be a composite since N mod(6)=3.

A very nice way to discuss primes geometrically is to use a hexagonal integer spiral first discovered by us a little over a decade ago. It places all integers , both prime and composites at the vertexes of a hexagonal spiral as shown-



The numbers where the spiral crosses the x axis are its vertexes at 6,12,18,24 and 30. Note that all primes five or greater are found along just the two radial lines corresponding to 6n+1 or 6n+5. Any number not lying along these lines must be composites. For more discussion on this hexagonal method of plotting integers we recommend a google search using the key words Morphing Ulam. This spiral approach for prime numbers has proven to be quite useful for discussing the properties of twin primes and giving the location of the vertex for any number. Thus N=6743 has N mod(6)=5. It therefore lies along the radial line 5-11-17-23-29. Sigma(N)-N=625, so the number is composite. It lies along 6n+5 at the 1123rd turn of the spiral.

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