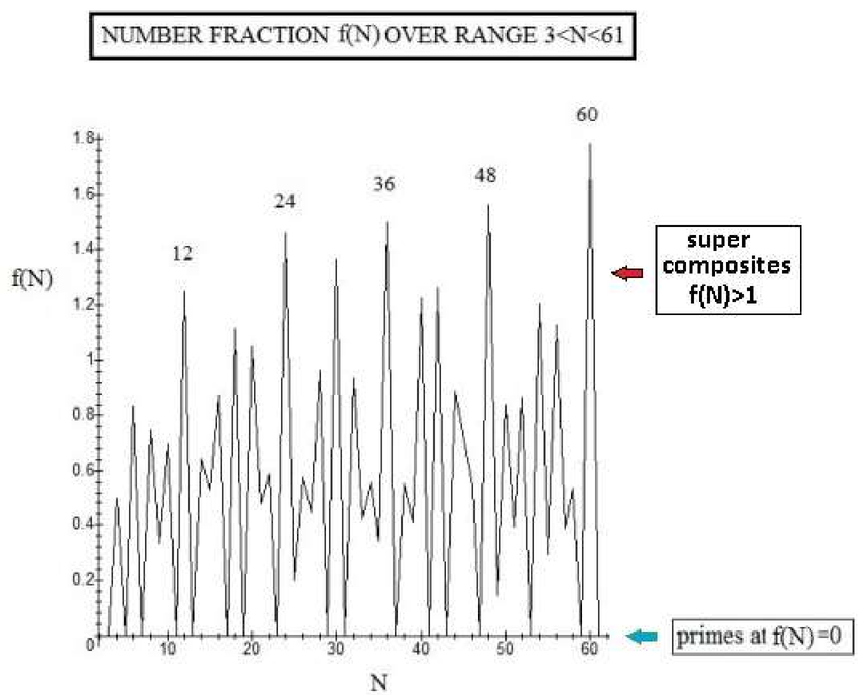
EVALUATION OF THE NUMBER-FRACTION FOR ANY INTEGER

About a decade ago (https://mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf) we came up with a new point-function which represents the ratio of all divisors of any integer minus the integer and one all divided by N. Mathematically one has-

$$f(N)=\frac{[\sigma(N)-N-1]}{N}$$

, where $\sigma(N)$ is the sigma function of Number Theory. This function, which we have called the number-fraction f(N), has the interesting property that it vanishes whenever N is a prime number. It exhibits rather wild fluctuations ranging from zero to, what we will show below, infinity. A plot of f(N) between N=3 and N=61 follows-



The average value in this range lies a little below unity with the primes clearly shown at f(N)=0. We call all those values of f(N) greater than one super-composites. We have found numerous applications for this function including its role in twin-primes, the construction of a general prime number function, and its role in factoring large semi-primes. It is our purpose here to determine the number fraction f(N) for any chosen integer N using both analytical approaches and computer evaluations.

We begin with finding all values for f(p^n), where p is any prime number. We have-

f(p)=0
f(p^2)=1/p
f(p^3)=(1+p)/p^2
f(p^4)=(1+p+p^2)/p^3

so that-

$$f(p^n)=1/[p^{(n-1)}]\sum_{k=0}^{n-2} p^k$$

Summing this last finite geometric series produces-

$$f(p^n)=\frac{1}{(p-1)}\left[1-\frac{1}{p^{n-1}}\right]$$

Thus f(p^n) approaches the finite value 1/(p-1) as n goes to infinity.
We have f(625)=f(5^4)=[1-1/125]/4=31/125=0.272.. . One can rewrite pf(p^2)=1 as a new prime number formula-

Any number satisfying J(N)=N/$[\sigma(N^2)-N^2-1]$ = 1/[Nf(N^2)]=1
is a prime number

To test this formula consider the number N=2861. It has-

J(N)=2861/(8188183-81855322)=1

So 2861 is a prime. For composite numbers, J(N) will lie in the range 0<J(N)<1. For primes we have J(N)=1.

If one has the semi-prime N=pq, its unique number-fraction is f(p,q)=(p+q)/pq. This lies close to zero as the primes p and q get large.

To calculate f(N) for any integer N up to about twenty digit length first go to-

http://www.javascripter.net/math/calculators/divisorscalculator.htm

to find σ(N) and then use f(N)=[σ(N)-N-1]/N.
So N=123456789 has $\sigma(N)$ =178422816 yielding f(N)=0.44522.. .

We find local maxima in the range up to 400 at N=

60=2^2x3x5 180=2^2x3^2*x5 360=2^3x3^2x5

Note that these numbers have the peculiar exponential form with the lowest prime components having the highest prime power. This suggests a new point function falling into the super-composite range having the forms-

2^3 x 3^2 =72 , 2^5 x 3^3 x 5^2=21600 etc.

So the prime components increase one prime at a time while their powers drop one prime at a time. This new rapidly growing point function generalizes to-

$$F= \prod_{k=1}^{m} ithprime(k)\hat{}ithprime(m+1-k)$$

The ithprime values are found, for example, at-

https://primes.utm.edu/nthprime/index.php#nth.htmlAs

As an example, we have for m=4 the large number-

F=2^7 x 3^5 x 5^3 x 7^2 = 190512000

This has –

$$\sigma(F) = 825355440$$

So that the number fraction becomes-

f(F)=(825355440- $190512000 - 1$)/190512000=3.33230158..

This f(F) lies clearly in the super-composite range. We have also evaluated f(F) for m= 10, 25, 50,100, and 150. In these cases we find-

f(10)=5.330946.. f(25)=7.311348.. f(50)=8.800312..

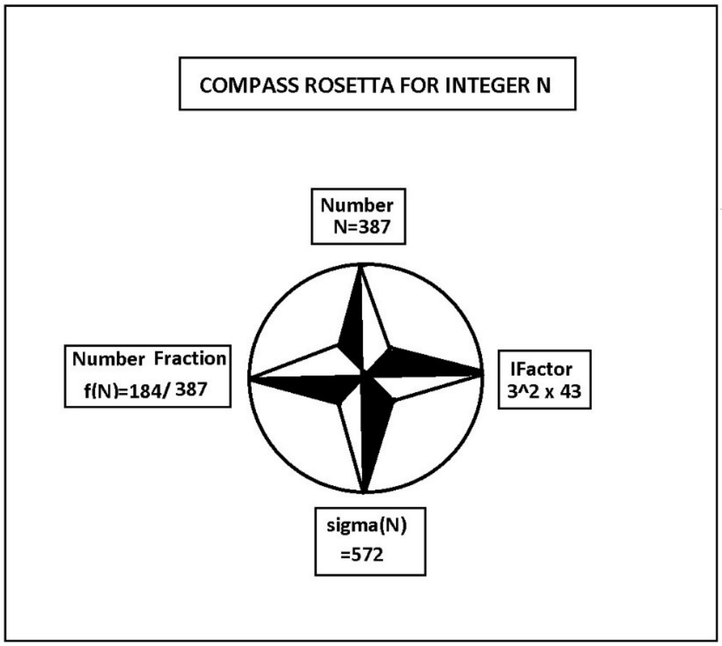
f(100)=10.267620.. and f(150)=11.115341..

Although my PC had no problems in finding f(F) for m less than 100, it did slow down to a four minute crawl finding the f(150) value of 11.115. In examining all of the above f(m) numbers, it seems that there is no limit to the super-composite value meaning the function –

lim m->∞ has f(F) infinite

It will, however , reach infinity very slowly somewhat similar to what happens with the harmonic series.

In conclusion we can say that the number-fraction can be evaluated exactly for all Ns less than 20 digit length using my PC. For Ns greater than this value a new analytic function F(m) can be used to calculate a new supercomposites f(F) out to infinite value. As a final thought, we can combine N-ifactor-sigma(N)-f(N) into the following four pointed compass rosetta-



U.H.Kurzweg
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