PARETO CURVES FOR WEALTH DISTRIBUTION

In 1906 the Italian engineer and economist Vilfredo Pareto proposed via statistical analysis that 80% of a countries wealth is held by about 20% of the people. This departure from an egalitarian state where each fraction of a population x owns the same fraction of wealth y was expressed by him in the mathematical formula-

$$ln(y) = ln(A) + nln(x)$$

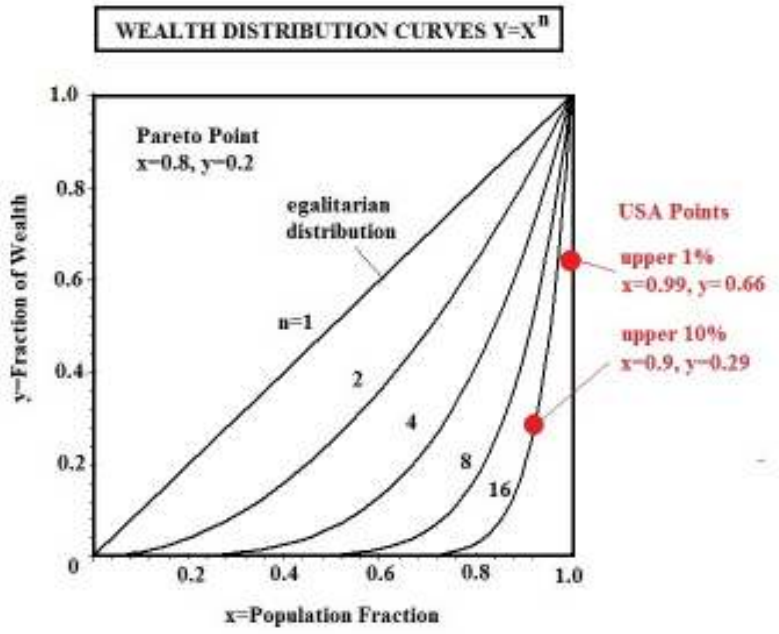
Here A is a constant. If one takes the exponent of this expression and demands that y=0 at x=0 and y=1 at x=1, there results the power law-

$$y = x^n$$

One knows from calculus that its derivative is $dy/dx=nx^{n-1}$ and, hence, y satisfies the first order differential expression-

$$\frac{dy}{dx} = \frac{n}{x}y \;\; subject\ to \;\; y(1) = 1$$

If we plot this function for values of n=1, 2, 4, 8, and 16, we get the wealth distribution curves shown-



In an ideal egalitarian state the curve corresponding to n=1 holds while in a totalitarian state where all power lies with just one person the n=∞ case applies. The curves in between refer to populations with different degrees of wealth distribution. In the United States the wealth breakdown is approximately given by the following table-

Percentile Fraction of the Population	Percent of Wealth Held
Upper 1%	34.3%
95%-99%	24.6%
90%-95%	12.3%
80%-90%	13.4%
60%-80%	11.3%
40%-60%	3.8%
0%-40%	0.02%

This distribution is seen to be highly slanted toward the very wealthy and is in excess of the 20-80 rule described by Pareto which corresponds to n=8. The upper 1% of the US population is seen to control about 34% of the wealth and the upper 10% controls a total of 71% of the wealth as indicated by the red dots in the graph. This distribution is thus seen to be close to the Paretto Curve $y=x^{16}$ with the very richest 1% requiring an even larger exponent of n=32.

As a measure of income inequality one can look at the ratio of the area between the n=1 curve and n=n curve versus the total area of ½ under the n=1 curve. Mathematically this ratio is expressed as-

$$G(n)=2\int_{x=0}^{1}(x-x^{n})dx=\frac{(n-1)}{(n+1)}$$

The ratio G(n)is often referred to as the Gini Coefficient. It's value is zero when there is perfect equality and 15/17=0.882 when n=16. It approaches a value of unity when n approaches infinity.

In the past governments treated such inequalities through graduated income taxes which take from the wealthy and distribute things to the poor. Also social programs such as social security, medicare and food stamps tend to reduce the value of n for a population. Without such pressure relieving programs, large income inequality can lead to protests such as those we are seeing in the present anti- Wall Street demonstrations, the riots in Greece and Spain and the French revolution of several hundred years ago. The dilemma facing the present state of this country is that the upper 1% have control of essentially all means of production plus control of all branches of the federal government, and control of the press. This makes the quantitative easing (QE1, QE2, Operation Twist, QE3?) possible despite of being opposed by

the majority of the population. Such easing benefits the well to do via increases in equity prices and bankers against derivative miscalculations, but hurts the rest of the population through increased inflation, near zero interest on savings, and larger deficits. The net effect will be to increase income inequality even further in the future. It is urgent that this country reduce the value of n to a more equitable value of n=8 . Some of the unbelievable prices we are seing presently for luxury homes, paintings and salaries of CEOs, movie stars, and athletes are a direct consequence of the trillions of dollars of fiat money which has been injected into the economy in the last four years by the Federal Reserve. It has not helped the average family whose real wealth has decreased by some 40% since 2008.