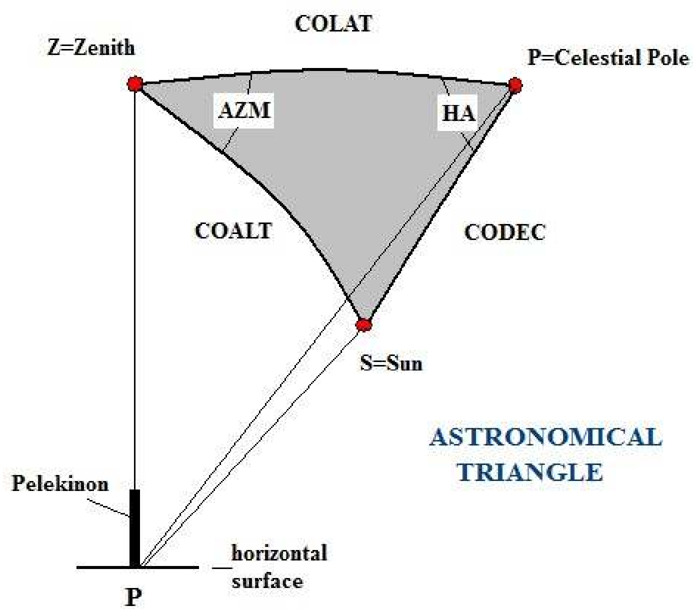
WHAT IS A PELEKINON AND HOW IS IT USED TO TELL THE TIME AND DATE

A Pelekinon is an early sundial used by the ancient Greeks to tell the hour of the day and time of the year by noting the angle and length of the shadow cast by a vertical rod placed perpendicular to a horizontal surface. The angle and shadow length were originally determined empirically. With the advent of spherical geometry it became possible to predict the shadow characteristics by evaluation of a few trigonometric formulas. We want here to look at the mathematics behind the Pelekinon and then apply the results to conditions here in Gainesville Florida at LONG 82.45W and LAT 29.65N.

Our starting point is the astronomical triangle formed on the celestial sphere. The triangle looks as follows-



The Pelekinon points straight up to the zenith Z and the sun's altitude above the local horizon is ALT=90-COALT. The latitude LAT=90-COLAT represents the angle above or below the equator.. The CODEC is the angle between the sun S and the celestial Pole P. This number will vary throughout the year. The azimuth AZM and the hour angle HA are two of the angles of the astronomical triangle as shown. From the Law of Sines for spherical triangles we have that-

$$\frac{\sin(AZM)}{\sin(CODEC)}=\frac{\sin(HA)}{\sin(COALT)}$$

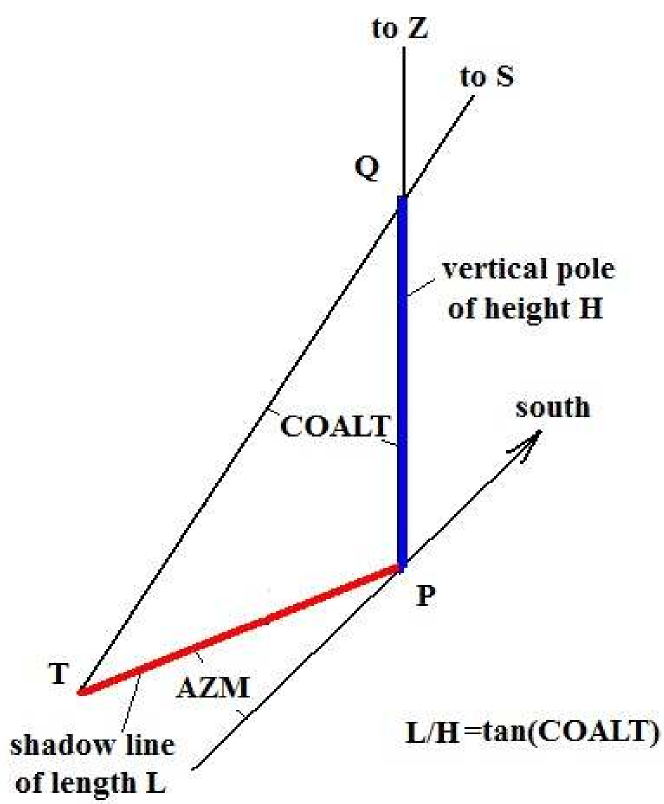
Also the Law of Cosines yields-

$$\cos(COALT) = \cos(CODEC)\cos(COLAT) + \sin(CODEC)\sin(COLAT)\cos(HA)$$

and-

$$\cos(CODEC) = \cos(COLAT)\cos(COALT) + \sin COLAT)\sin(COALT)\cos(AZM)$$

We need to solve these three equations for HA and COALT to determine exactly the angle the shadow makes with respect to the north-south line and its lengt L The geometry we are dealing with for the Pelekinon looks like this-



Solving the above three equations and making use of the geometry shown, one finds-

$$\frac{L}{H} = \tan\{\arccos[\sin(LAT)\sin DEC) + \cos(LAT)\cos(DEC)\cos(HA)]\}$$

and-

$$\sin(AZM) = \frac{\cos(DEC)\sin(HA)}{\cos(ALT)}$$

At local noon the HA=0 and these simplify to-

$$AZM = 0 \quad and \quad \frac{L}{H} = \tan(LAT - DEC)$$

Thus, on the equator , there is no shadow at local noon during the equinox but there will be at other times of the year. Sunrise and sunset occur when ALT=0 or COALT=90. So the cosine of the hour angle becomes-

$$\cos(HA) = [-\tan(DEC)\tan(LAT)]$$

On the equator the HA at sunrise is always 6 hours before local noon.. Above the arctic circle located at LAT 66.5 degrees the sun will not rise at all at winter solstice on Dec.21. Here in Gainesville(LAT=29.65) the sun rises at HA= 7.246 hours before local noon during the summer solstice on June 21 . We actually have less daylight than places north of us at that time. For example, Washington DC (LAT= 38.88) will have a total of 15.55 hours of sunshine on the same June day. I remember as a youngster, while being brought up in the DC area, that in June we had days where it would not get dark till about 9pm daylight savings time.

Let us next calculate the position T that the tip of the shadow created by the vertical rod throughout the day and year. One has, in terms of polar coordinates (r,θ) that-

$$r = \frac{L}{H} = \tan(COALT) = \frac{1}{\tan(ALT)} \quad and$$

$$\theta = AZM = \arcsin\left\{\frac{\cos(DEC)\sin(HA)}{\sin(COALT)}\right\}$$

Substituting in for r and θ, there follow from the above given law of sines and cosines the three equations-

$$\frac{1}{\sqrt{1+r^2}} = \sin(LAT)\sin(DEC) + \cos(LAT)\cos(DEC)\cos(HA)$$

$$r\sin(\theta) = \sqrt{1+r^2}\cos(DEC)\sin(Ha)$$

$$\sqrt{1+r^2}\sin(DEC) = \sin(LAT) \pm r\cos(LAT)\cos(AZM)$$

We can combine these equations to find-

$$1 + x^2 + y^2 = \left\{\frac{\sin(LAT) - y\cos(LAT)}{\sin(DEC)}\right\}^2$$

and

$$\left\{\frac{\dfrac{1}{\sqrt{1+x^2+y^2}} - \alpha}{\beta}\right\}^2 + \frac{x^2}{1+x^2+y^2} = \gamma^2$$

when expressed in Cartesian coordinates x=rsin(θ) and y=rcos(θ). Here the known constants are-

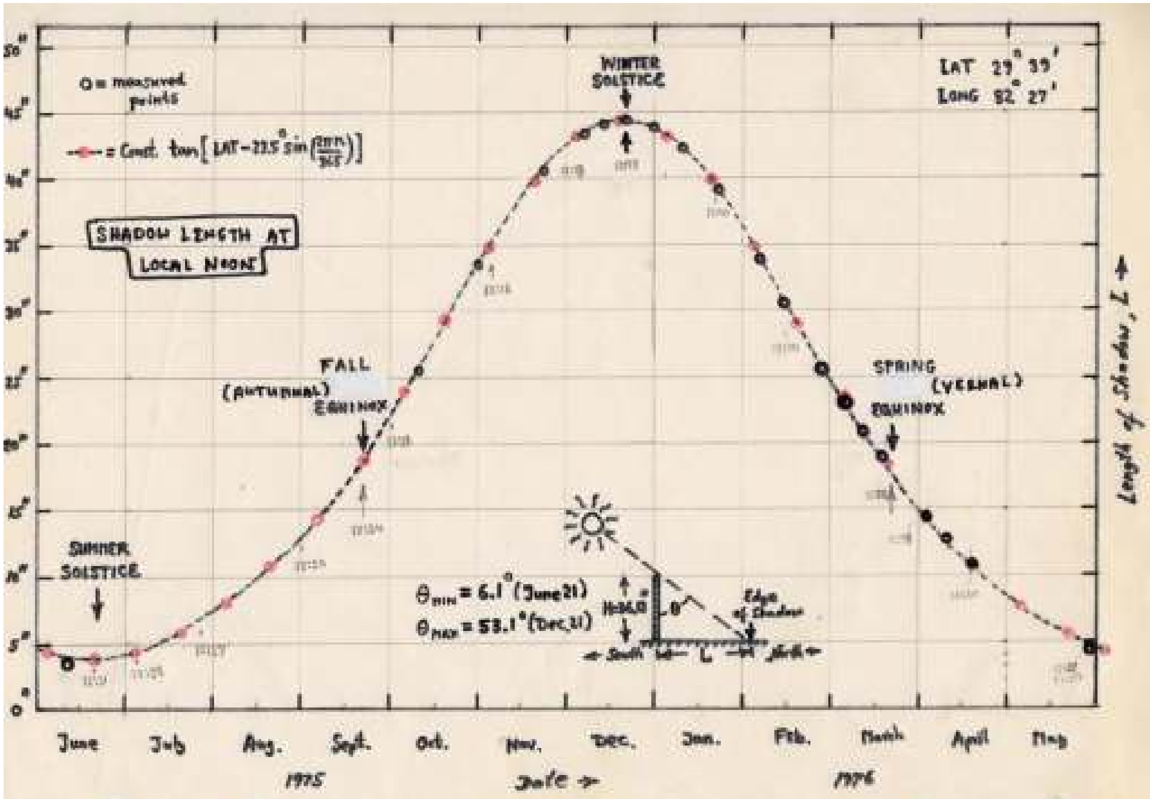
$$\alpha = \sin(LAT)\sin(DEC), \quad \beta = \cos(LAT), \ \gamma = \cos(DEC)$$

These are rather complicated expressions which simplify considerably in certain limiting cases. One sees that at local noon x=0 and y becomes-

$$y = \tan(LAT - DEC)$$

So during the equinoxes ,where DEC=0 , y simply equals the tangent of the lattitude. At noon here in Gainesville, FL one will observe the tip T of the shadow go from y=L/H=1.3342 at the winter solstice to 0.1077 at the summer solstice. It is located at y=0.56923 at the equinoxes.

Quite a few years ago I built a simple Pelekinon in my backyard and recorded the local noon positions of T throughout the year. A scan of a graph of these observations follows-



The agreement between actual measurements and the above formula y=tan(LAT-DEC) is seen to be excellent. We used the approximation that the number of days n after the equinox is given by-

$$n = \frac{180}{\pi}\arcsin\left(\frac{DEC}{23.5}\right)$$

More precise values for DEC are obtainable by referring to a nautical almanac.

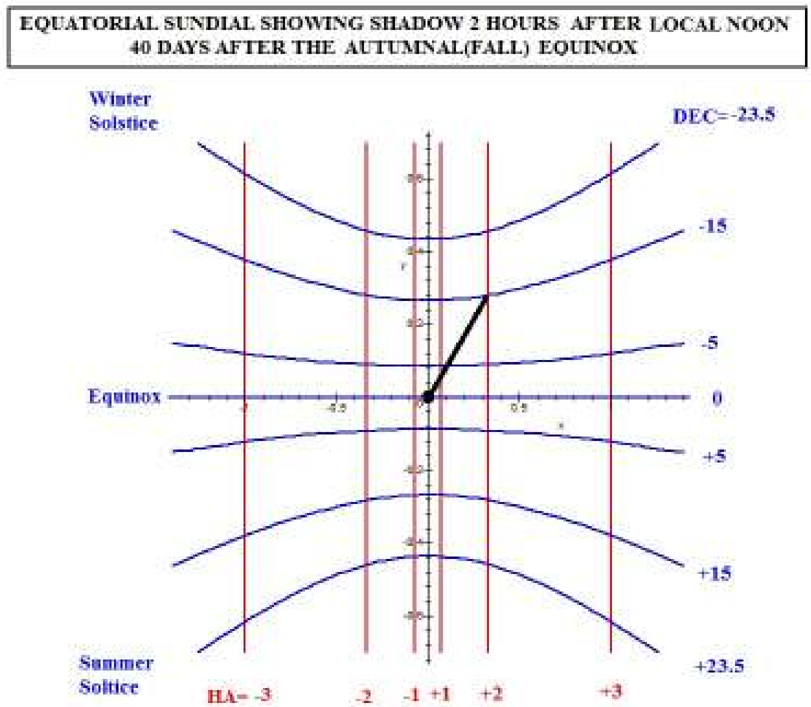
If the LAT becomes zero by putting the Pelekinon on the equator, then things simplify considerably. Now one finds, after a little manipulation, that the tip of the shadow T traces out the symmetric hyperbolic pattern given by-

$$y^2[\cot(DEC)^2]^2 - x^2 = 1$$

The maximum and minimum values of y when x=0 are given by ±0.434812. The corresponding hour angle lines lie parallel to the y axis and are given by-

$$x = \tan(HA)$$

A plot of the movement of the shadow according to the last two equations looks as follows-



In this figure we show the shadow two hours after local noon about 40 days after the fall equinox. Note how the shadow will move in a clockwise manner throughout the day. This is undoubtedly why the dials on our watches move in the direction they do. The first pocket watch makers(Heinlin, Nuremberg Eggs 1505) were imitating the behavior of sundials .

Note that this HA angle independence on y will disappear when moving to a LAT different from zero. One can construct sundials away from the equator by tilting the Gnomon and the ground plane at an angle equal to the local latitude LAT . Sundials of this type are known as equatorial sundials. To read them throughout the year on needs to look at the two sides to the ground plane. Here is a photo of an ancient Chinese equatorial sundial-

The Gnomon is pointed toward the north star (celestial pole) and the two sides of the stone disc are market by empirically determined curves allowing the determination of the hour (HA) and number of days n away from the equinoxes.