HOW DOES ONE DETERMINE WHETHER A NUMBER IS PRIME OR COMPOSITE?

As we all learned in junior high school there are two types of integers. One set is referred to as the **composites** and is characterized by having several divisors in addition to one and itself. The second group are the **primes** which can be divided by only one and the number. The earliest people who showed interest in these two groups of integers where the ancient Greeks (Pythagoras and Plato in 500BC and 400 BC, respectively), and especially the mathematicians at the Greek School in Alexandria, Egypt established about 300BC . The Greek School and Library had on its faculty at one time or another Euclid, Archimedes, Diophantus, Hypatia, and Eratothenes. The library was burned down in 642 AD during the Muslim conquest.

Eratosthenes(276BC-195BC) was the first person to propose an algorithm for distinguishing composite from prime numbers. Now known as the **Sieve of Eratosthenes**, it starts with writing down the integers in ascending order starting with two as follows-

2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,…

One next divides every integer by 2 and striking out those numbers were a division is possible. Then one repeats the procedure with 3, striking out any number which is divided by it. Continuing the process with 5, then 7 and so on, one is left with the following residue

2, 3, 5, 7, 11, 13, 17, 19, 23, 29,

These represent the lowest ten prime numbers. The discarded set reads-

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30

and represents the composite numbers. One notes at once that all the prime numbers with the exception of 2 are odd numbers while any even number is composite but many odd numbers are also. Although the Sieve of Eratothenes works fine for distinguishing prime from composite numbers for smaller N it becomes quite cumbersome when determining the character of large integers such as-

$$N = 4512851897$$

Is this number prime or composite? To answer this question we make use of what we have recently found and reported in two pdf files found at the bottom of our mathematics page located athttp://www2.mae.ufl.edu/~uhk/MATHFUNC.htm

We showed there that (1) all prime numbers above N=3 are of the form Q=6n+1 or Q=6n-1, where n are positive integers, and (2) a composite number is characterized by having its number fraction different from zero. The number fraction is defined as-

$$f_N = \frac{(sum\ of\ all\ \ factors) - (N+1)}{N}$$

So if we take the composite number N=12 we have the factors 1, 2, 3, 4, 6, and 12. This means f_{12}=[(1+2+3+4+6+12)-(1+12]/12=1.25. On the other hand if we look at the prime number N=13, you get f_{13}=(1+13)-(1+13)]/13=0. From this we see that an f_N equal to zero corresponds to a prime number and any value different from zero defines a composite number. Furthermore when one plots f_N versus N it is found that the all numbers which are multiples of six have the largest number fractions in their immediate neighborhood and **all primes above N=3 lie within one unit of such multiples of six**. Thus N=6 x 3 +1=19 and 6 x 3-1=17 meet the condition for primeness. However, since often an odd number 6n±1 will not be prime, one must also evaluate the number fraction f_N . For the number N=4512851897 given above we have-

(N+1)/6= 752141983 and (N-1)/6=2256425948/3

This means N may possibly be prime since the number N can be written as 6(752141983) -1. However a computer evaluation of the number fraction for N yields-

f_N=**0 .03282590**

The evaluation involves the simple one-line MAPLE command-

with(numtheory): evalf((add(i,i=divisors(N))-(N+1))/N);

Thus the number is composite. Indeed a non-required evaluation shows that-

4512851897=53 x 73 x 1166413

Sometimes the determination if a number is prime is even easier. Take the case of-

N =2752326559594673596887 57

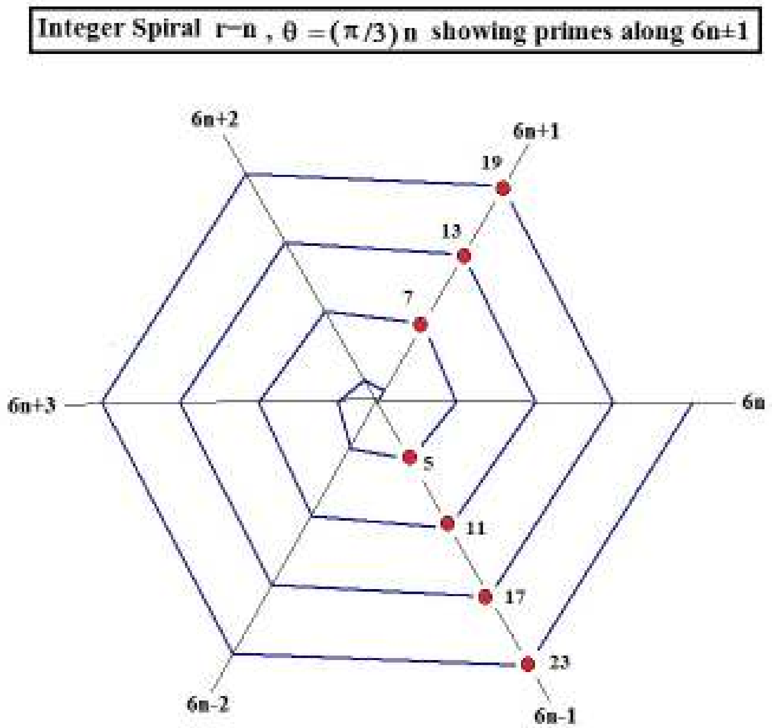
Here the divisions (N+1)/6 and (N-1)/6 do not yield integer values, meaning that the condition for primeness is violated and hence N is composite.

Finally let us run through all the primes given by Q=6n±1 produced for n=1 through 12. The results are summarized in the following table-

n	Q=6n-1	Q=6n+1
1	5	7
2	11	13
3	17	19
4	23	-
5	29	31
6	-	37
7	41	43
8	47	-
9	53	-
10	59	61
11	-	67
12	71	73

The results reproduce all primes above N=3 up to N=79. The few places marked blank indicate location of those Qs which are not primes. From this we can state that the Q numbers are a necessary but not sufficient condition for the existence of a prime number. One can confirm that the numbers located in the blank spaces are composite by carrying out the operations Q/ithprime, where the ithprime equals 5, 7, 11, 13, etc and showing that one of these divisions yields an integer. Thus Q=6(13)-1=77 is not prime since Q/11 is an integer.

We have recently found an interesting way to plot the prime numbers along two lines intersecting a hexagonal spiral. Here is the picture-



The red dots indicate the lower primes above N=3. Notice that all the odd numbers lying along 6n+3 such as 15, 21, 27, etc. can never be primes.