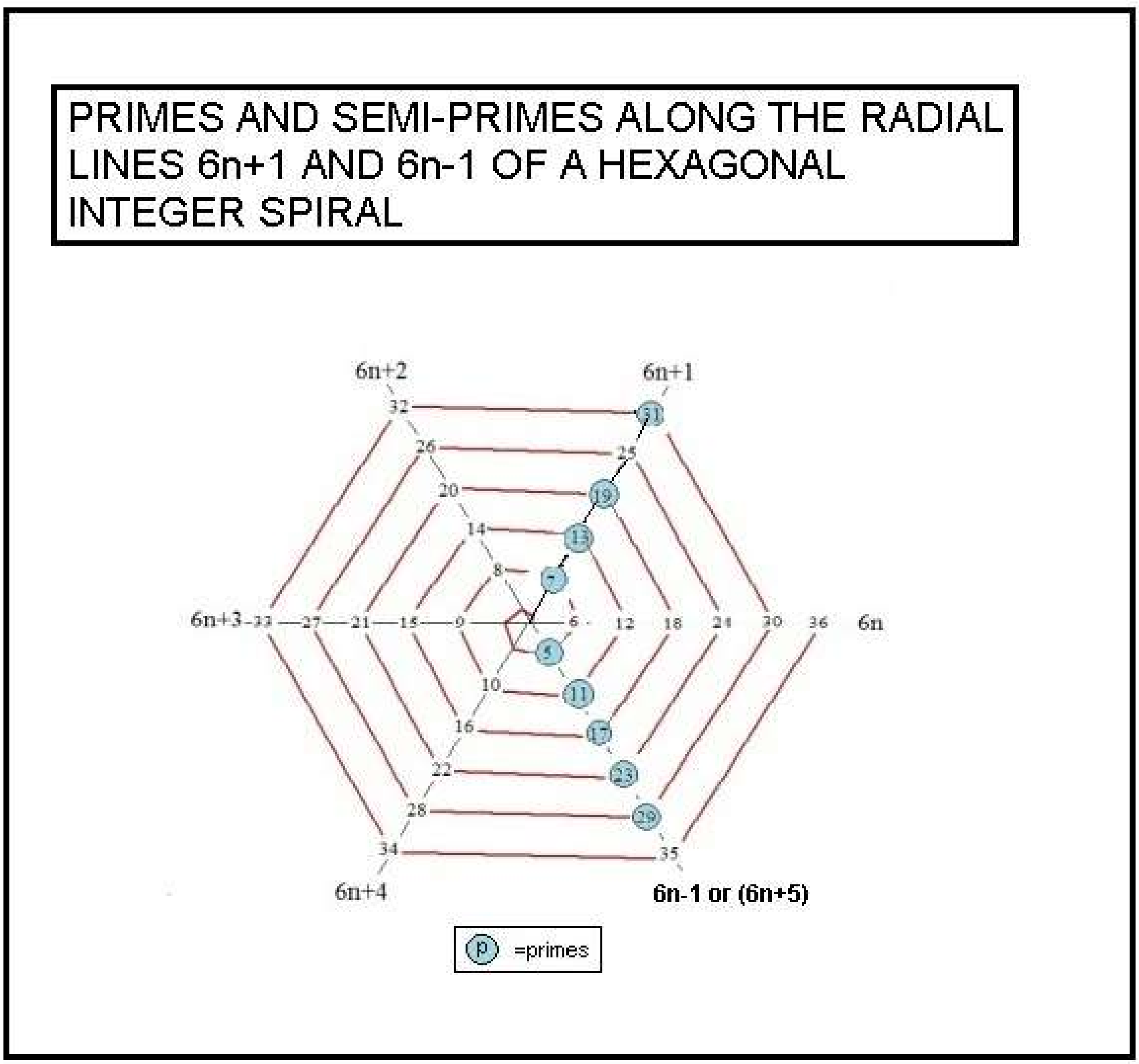
THE HEXAGONAL INTEGER SPIRAL AND ITS CONNECTION TO PRIME NUMBERS

Some fifteen years ago we first showed that the standard Ulam Spiral can be converted to a very simple form in which all primes five and greater are located at the intersection of the vertexes of a hexagonal integer spiral and two sixty degree radial lines $6n\pm1$. (see https://mae.ufl.edu/~uhk/MORPHING-ULAM.pdf). The resultant picture looks like this-

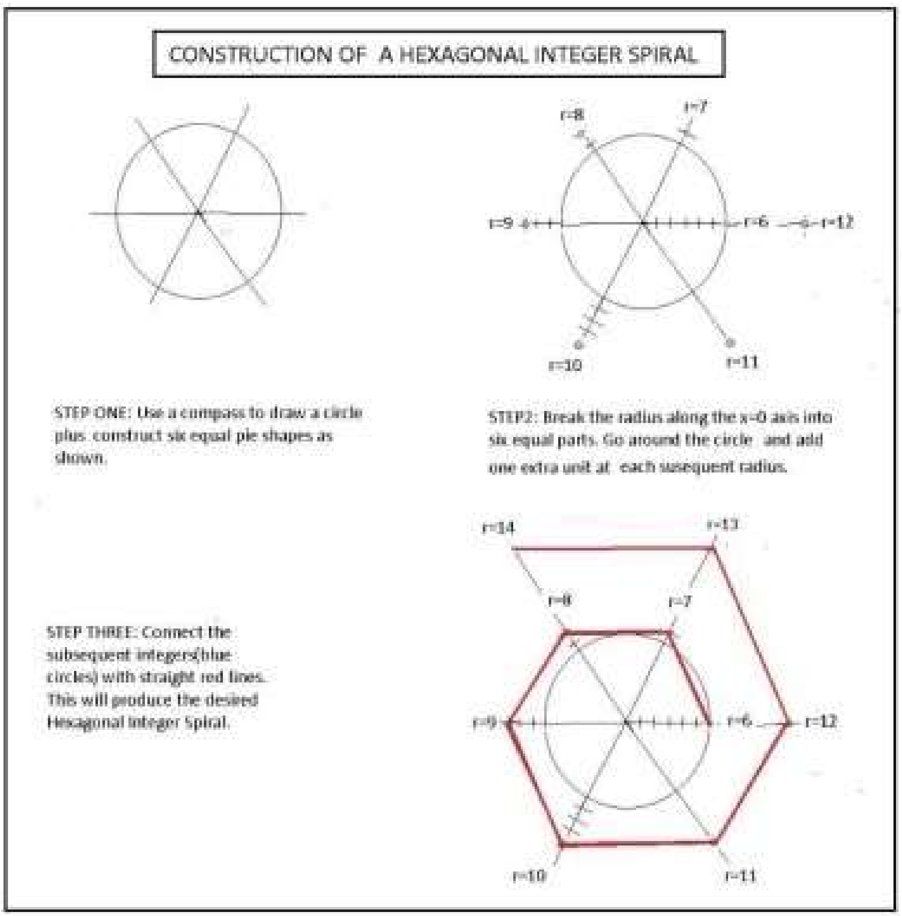


From this result we have the observation that –

A necessary but not sufficient condition that a number N , five or greater, is prime is that it satisfies $6n\pm1$.

The two radial lines can also contain numbers which are composite such as N=25 and N=35. It is our purpose here to examine this Hexagonal Integer Spiral in more detail and review how it can be used to discover further details involving prime numbers.

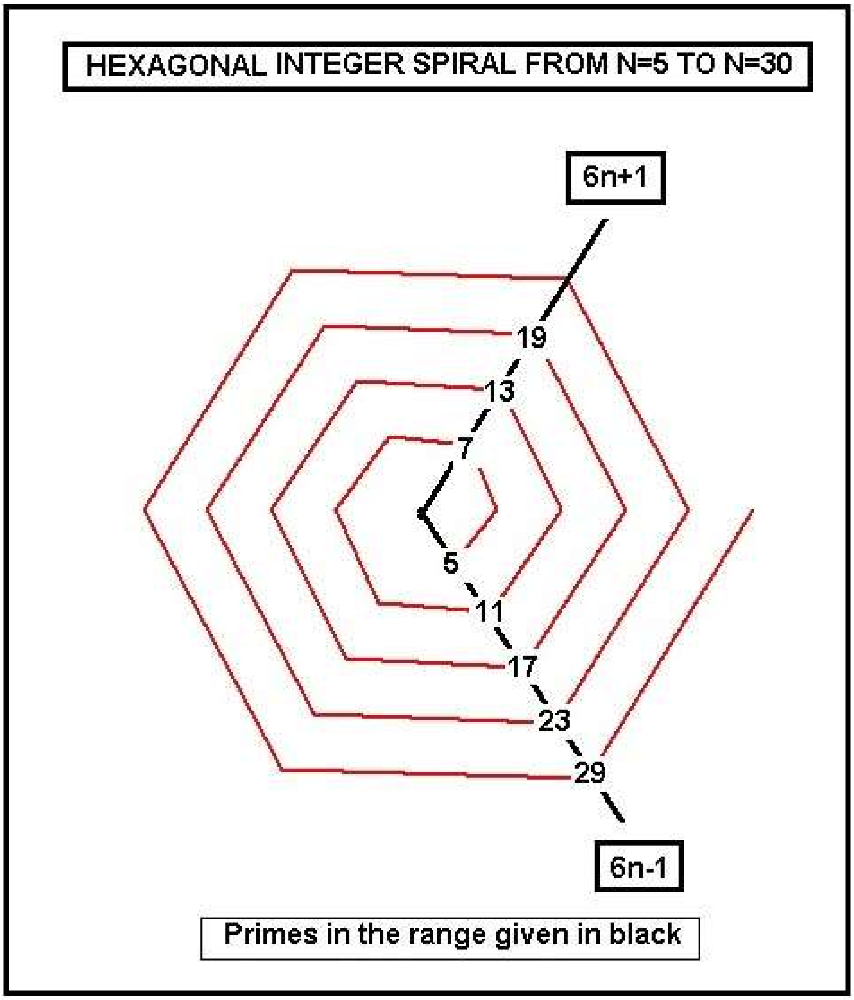
Our starting point is to draw a hexagonal spiral having six vertexes per turn. This can be accomplished geometrically as shown in the following-



We can also carry out the computer operation-

```
with(plots);   listplot([seq([n*cos(n*Pi/3),n*sin(n*Pi/3)],n=5..30)],thickness=2, color=red, scaling=constrained, axes=none);
```

It produces the Hexagonal Integer Spiral shown-



We have added the two radial lines $6n\pm1$ along which the indicated primes N are found. Note that the jump in value from one turn of the spiral to the next is always 6. There are some points along these prime locations where the number N is composite such as N=25 or 35. This means that we can only make the more restrictive statement that-

A necessary but not sufficient condition for N five or greater to be prime, is that it must satisfy $6n\pm1$ or the equivalent number theory condition that N mod(6)=1 or 5.

We first came up with the final form shown in figure one above back in August of 2008. Since that time I have been using this new approach to find some new relations involving primes. One of the more interesting of these is the location of twin primes [5,7], [11,13], [17,19] etc. These all have a mean value of 6n. So, for instance, the twin prime [59,61] has a mean value of 60=6(10) and the twin prime [857,559] has a mean value of 858=6(143). There probably are an infinite numbers of twin primes, although this conjecture needs to still be proven. Note that our Hexagonal Integral Spiral shows that there can be no three primes differing from each other by 2 each. However the three prime possibility of $[p_1,p_2=p_1+2,p_3=p_2+4=p_1+6]$ exists. Two examples are [11,13,17] and [17,19,23].

Consider next a semi-prime N=713=pq , where sqrt(N)=26.70. Here N mod(6)=5 so that N =6(119)-1 . It produces-

$$(6n+1)(6m-1)=6(119)-1$$

This is equivalent to-

$$m=(119+n)/(6n+1)$$

It solves as n=5 and m=4. So we get the prime number solutions-

$$713=31x23$$

Note that 23<sqrt(713)=26.70<31. It is to be expected that one of the primes falls within a circle of radius sqrt(713) and the other outside the circle.

Finally we note that one can determine whether the number lying along either 6n+1 I or 6n-1 is prime or composite by looking at its sigma. For primes $\sigma(N)=N+1$, while for a composite this will not be the case. Let us demonstrate for N=25. We have $\sigma(25)=31$ and so this number is composite while for N=79 we have $\sigma(79)=80$ so 79 is a prime.

U.H.Kurzweg
February 5, 2024
Gainesville, Florida