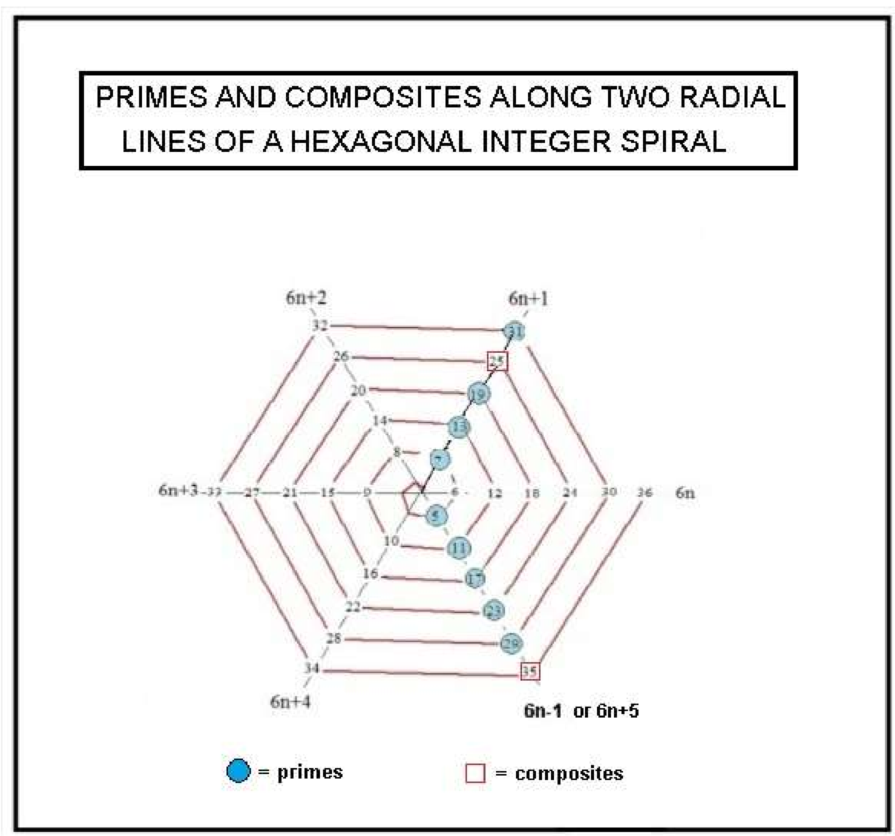
DETERMINING WHETHER A NUMBER IS COMPOSITE OR PRIME

In earlier articles on this Tech-Blog Page we have described several new ways to determine whether a number is prime or composite. Summarizing these observations we have found that the simplest criteria for primeness is –

$$N \bmod(6)=1 \text{ or } 5 \text{ with } \sigma(N)=1+N$$

, provided N is five or greater and σ(N) is the sigma function of number theory representing the sum of all its divisors. Geometrically one can show that this criterion places all primes along just two radial lines 6n±1 at the vertexes of a hexagonal integer spiral as shown-



To test out this criterion let's look at some specific examples.

Starting with-

N=578104329 where N mod(6)= 5 and σ(N)=5782104330

, we conclude at once that this is a prime. Next take-

N=835621891 with N mod(6)=1 but σ(N)=899900512 .

There σ(N)-N is greater than one, so N must be a composite. Integers lying at all other vertexes of the integral spiral are composite numbers such as –

N=2247537 where N mod(6)=3

.Notice the above primeness criterion does not require explicit values for the prime components of a composite. Almost three hundred years ago, Leonard Euler proved, after lengthy calculations, that the 5th Fermat Number N=2^32+1 is a composite although Fermat had thought it to be prime. The above criterion, shown in blue, readily confirms that this 5th Fermat Number is a composite.

To find the explicit value of the prime components of any positive integer N, we start with the well known identity -

N=(p^a)(q^b)(r^c)…

, with p, q, r,.. being increasing prime values taken to the integer powers a, b, c etc. A simple version of this form is the semi-prime N=pq for which-

σ(N)= σ(p)σ(q)=(1+p)(1+q)=1+(p+q)+N

Hence-

p+q= σ(N)-N-1 and pq=N

Eliminating q from these two equalities produces the quadratic

p^2-p{σ(N)-N-1}+N=0

for any semi-prime N=pq. The two roots of this last equation yield the values of the primes p and q. So, for example, N=77 has σ(N)=96. This produces p=7 and q-=11.

When N contains more than two primes, one no longer finds simple closed form solutions involving prime factors. However, with the use of any advanced mathematics programs, such as Maple or Mathematica, one can get the prime components directly. Take the example of –

$$N=548207 \quad \text{with} \quad N \bmod(6)=5 \ \text{ and } \ \sigma(N)=654720$$

This is a composite. Performing the built-in ifactor operation we get-

$$\text{Ifactor}(548207)=11 \times 19 \times 43 \times 61 .$$

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