

MASS , SPEED, MOMENTUM AND ENERGY OF PROJECTILES

The other day while I was waiting for my car to be fixed at our local Pep-Boys I ran across some old hunting magazines in their waiting room. Being a non-hunter I was not familiar with certain terms like draw-weight and grain mass as mentioned in the magazine articles although I am quite familiar with velocity , mass , momentum, and kinetic energy concepts through my earlier teaching and applied science career as a university professor. The articles in these magazines got me to thinking about expressing the four concepts of mass, velocity, momentum and kinetic energy for a variety of different projectiles in terms of the MKS system.

Before getting into the specifics for different projectiles, we first define the above mentioned concepts in generic form. We present these in MKS form and also add the needed conversions to decipher the terms used by hunters. They are-

Velocity: $1 \text{ m/s} = 3.281 \text{ ft/sec}$

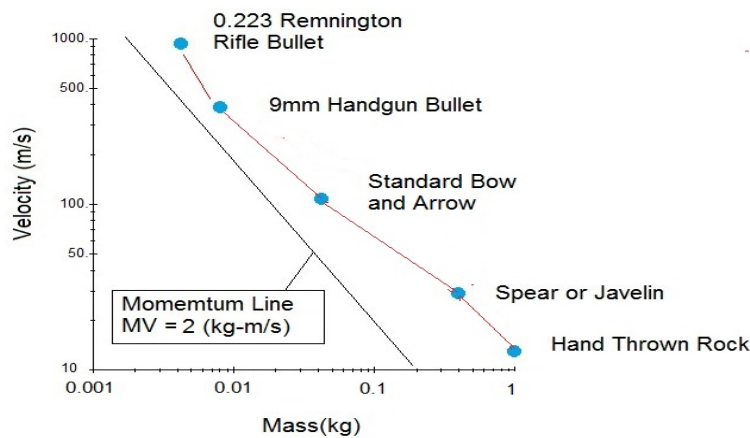
Mass: $1 \text{ kg} = 0.0685218 \text{ slugs} = 15432.57 \text{ grains}$

Momentum: $1 \text{ kg-m/s} = 1 \text{ N-s} = 0.22482 \text{ slug-ft/sec}$

Kinetic Energy: $1 \text{ kilogram-meter}^2/\text{sec}^2 = 1 \text{ joule} = 0.737562 \text{ ft-lb}$

With these basic definitions and their conversions we can now examine different projectiles. For this purpose I have chosen to lookup the characteristics of five different projectiles. These are plotted as launch velocity versus the projectile mass in the following loglog graph. The blue points are averaged values of the information obtained from the internet-

MASS AND VELOCITY OF VARIOUS PROJECTILES



It is quite clear from this graph that velocity increases with a decrease in mass of the projectile. Early man had available as his weapon projectiles speed which did not exceed 100m/s. It was not until the invention of gun-powder that hand-held devices were able to reach speeds which now can top values of 1000 m/s. You will note that the momentum of these projectiles all lie a little above 2 kg-m/s. Since momentum gives a good measure of kill power it is seen that a hand thrown rock can be just as lethal as a rifle bullet when applied under the right conditions. The advantage of high velocity launch velocities is that they allow a much wider kill range.

Hunting magazines typically describe the projectiles for bow and arrow and rifles in terms of their kinetic energy. They express this energy in terms of foot pounds (ft-lb). In terms of MKS units the kinetic energy $KE = MV^2/2$ is described in terms of joules= Newton-meter. It also can be expressed as $kg-m^2/s^2$. Here is a table giving the launch kinetic energy for the five different projectiles whose characteristics are shown in the above graph-

Projectile from:	Kinetic Energy in $kg-m^2/s^2$
Remington 0.223 Rifle	1672
9mm Hand Gun	541
Spear or Javelin	270
Bow and Arrow	95
Hand Thrown Rock	72

I have placed the projectiles in descending order of their kinetic energy. Clearly those projectiles with velocities produced by gun powder exceed those possible by kinetic arm action. If one wants to have projectiles with kill abilities at large distances, the rifle with large cartridges but small caliber bullets is the preferred weapon. One knows that in a vacuum the maximum range R of a projectile goes as-

$$R = \frac{v^2}{g} \sin(2\theta)$$

so that it equals the square of the launch velocity v times the sine of twice the launch angle θ divided by the acceleration of gravity. Air resistance will reduce this range by a calculable amount, but it does not take away from the fact that R will be a function of velocity squared. For the Olympic javelin throw one has a mass of 0.8kg launched at $v=70\text{mph}=31\text{m/s}$ at an angle slightly less than 45 degrees. The maximum predicted range using the above formula happens to exactly match the $R=98\text{ m}$ Olympic record.

For a sniper rifle such as the Army M24, the bullet has a mass of 170 grains=11gm=0.011 kg. Its muzzle velocity is 816 m/s and it has an effective kill range of 1000 m. With air-drag it will take the bullet a little more than one second to reach the 1000 m range. During that time to the bullet will drop about $y=0.5gt^2 \approx 5\text{m}$ so adjustments need to be made by elevating the rifle barrel to-

$$\theta = \frac{1}{2} \arcsin\left(\frac{Rg}{v^2}\right) = 0.4220 \text{ degrees}$$

relative to the horizontal. For super-accurate shots the sniper must also know the exact distance to the target obtained by a laser range finder but also some knowledge of wind direction.

We have examined the characteristics of various projectiles giving their momentum and kinetic energy in MKS units. In all cases their momentum lies slightly above 2 kg-m/s with the widest effective kill range increasing with increasing projectile speed. Future developments for even more deadly hand-held weapons should involve rifles with longer barrels and use of more chemical energy stored in one or multiple cartridges in order to increase chamber pressure. Also other propulsion means such as electromagnetic drives should be investigated.

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