

## SANDWICH STRUCTURE OF ALL TWIN PRIMES BORDERING A CENTRAL SUPER-COMPOSITE

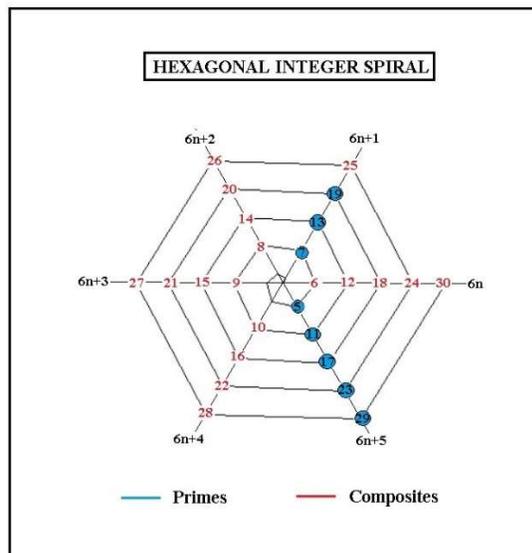
If one writes down the first twenty integers in ascending size, starting with two, one gets the following array-

$$S = \{ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \ 21 \}$$

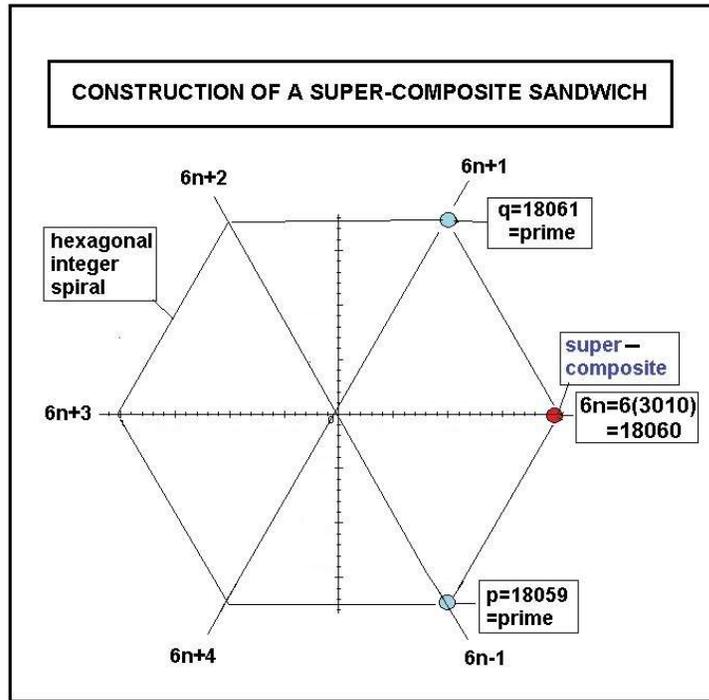
On inspecting this array, one notices certain integer triplets of a prime followed by a composite and then another prime. Examples are-

$$3-4-5 \quad 5-6-7 \quad 11-12-13 \quad 17-18-19$$

We will call these three integer groupings *integer sandwiches*. The surrounding primes here are known as twin primes. The central composite number must always have the form,  $N=6n$  with  $n$  chosen to produce primes  $q=6n+1$  and  $p=6n-1$ . The following hexagonal integral spiral (first found by us about a decade ago) shows why twin primes must always be symmetric about the number  $N=6n$ .



Primes about  $N=6n+3$  have the form  $p=N-2$  and  $q=N+2$  and thus no longer can produce standard twin primes. One way to rapidly find integer triplets is to use the following geometrical construction method-



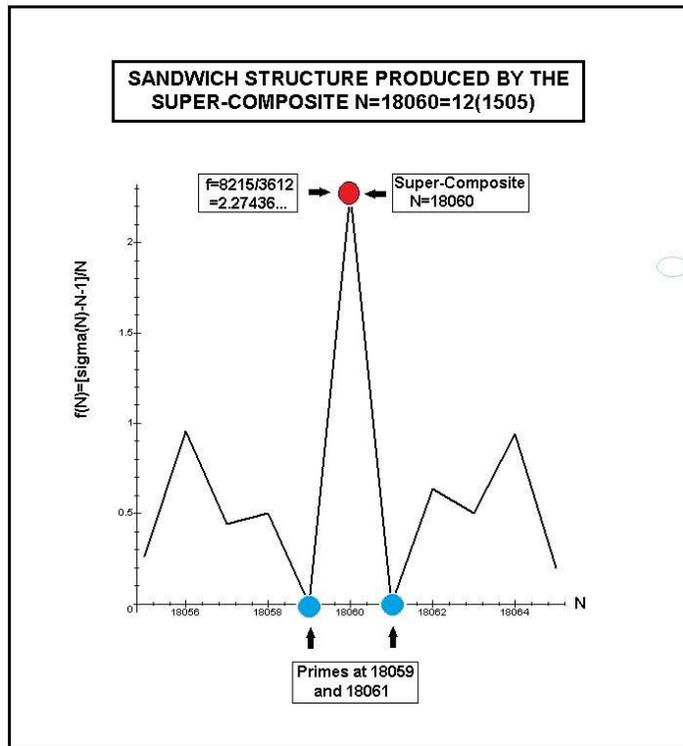
Here we have used  $N=18060=6(3010)$ . Note that this number produces the integer triplet sandwich-

$$\begin{array}{ccc}
 18059 & - & 18060 & - & 18061 \\
 \text{(blue)} & & \text{(red)} & & \text{(blue)}
 \end{array}$$

An even quicker way to generate an integer triplet is to make use of the number fraction defined as follows-

$$f(N) = \left[ \frac{\sigma(N) - N - 1}{N} \right]$$

Here sigma is the sigma function of N with the numerator of f just designating the total sum of the divisors of N. When  $f=0$  we have N being a prime and when  $f>1$  N is a super-composite having many divisors. Plotting f versus N near 18060 produces the picture-



Clearly this  $N$  versus  $f(N)$  plot makes the finding of an integral triplet very easy. One begins by plotting  $f(N)$  over a limited number range and then checks those numbers where  $f$  is greater than unity. One next checks the value of  $f$  at  $6N+1$  and  $6N-1$  about such a super-composite. If they both are zero, the integer sandwich has been found. Otherwise one keeps searching using a shifted integer range.

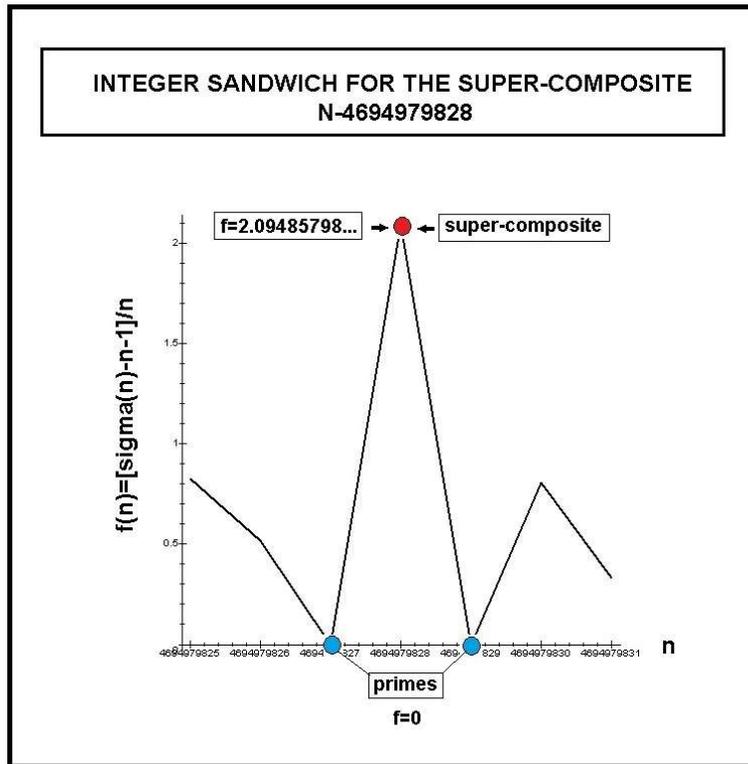
Let us demonstrate the use of the  $f$  method for the large ten digit long semi-prime-

$$N = 6(782496638) = 4694979828$$

Here we find  $f(N) = 2.09485798\dots$ . Next looking at  $f(N+1)$  and  $f(N-1)$ , we find these to be zero. Hence we have a new integer sandwich-

$$\begin{array}{ccc}
 4694979827 & - & 4694979828 & - & 4694979829 \\
 \text{prime} & & \text{super-} & & \text{prime} \\
 & & \text{composite} & & 
 \end{array}$$

A graph of  $f(n)$  versus  $n$  in the neighborhood of  $N$  yields the following-



The graph looks very much like the earlier one for  $N=18060$  with a large central peak(the slice of meat) surrounded by zero  $f$  value (the slices of bread).

A final point I wish to make concerns the quick finding of super-composites. These are easiest to generate by noting that the prime factors must contain the powers of mainly the smallest primes. This suggests we can find super-primes using the product expressions-

$$S=2^a 3^b 5^c 7^d \dots \quad \text{with } a>b>c>d$$

Consider  $S=(2^{15})(3^8)5^3=26873856000$ . This number is divisible by six suggesting the possibility of primes for  $S+1$  and  $S-1$ . Doing a prime test shows indeed that these two numbers are prime and we get the integer sandwich-

$$2687385599 - 26873856000 - 26873856001 \quad .$$

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**ps-The word sandwich has an interesting origin. It seems that John Montagu(1718-1792) also known as the 4<sup>th</sup> Earl of Sandwich was an avid gambler. To not be disturbed by leaving for lunch while gambling, he had his servant prepare and then deliver to the gambling table several slices of meat held together by opposing slices of bread. This new food configuration soon caught on throughout the kingdom and a new name was born. Although the Earl of Sandwich is mainly remembered today for the sandwich episode, he did accomplish many other things especially as Lord of the Admiralty and for service in the House of Lords in Parliament. Captain Cook even named the Hawaiian Islands after him.**