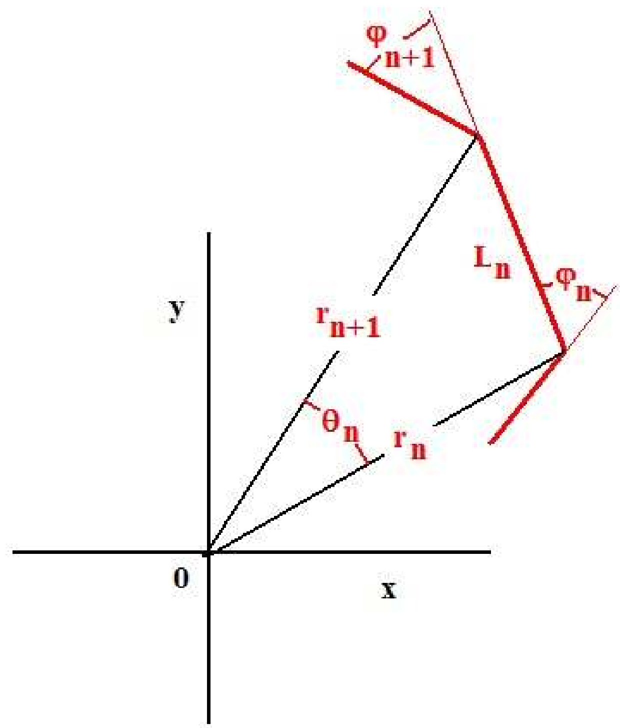
SPIRALS CONSTRUCTED FROM STRAIGHT LINE SEGMENTS

If you take a straight line of infinite length and break it up into line segments L_1, L_2, L_3 , etc. and rotate segment L_n relative to L_{n-1} by angle φ_n counterclockwise you will construct a figure, which under the condition that the radial distance r_n from the origin to corner between L_n and L_{n+1} is always less or more than r_{n+1} the result will be a spiral. The definition sketch of such a spiral looks like this-

DEFINITION SKETCH FOR SPIRAL GENERATION



For purposes of the discussion below we will always take the first line segment L_1 to start at the origin and have unit length. In defining the spiral one can either state how r_n varies with angle θ_n or give segment length L_n as a function of bend angle φ_n. From the law of cosines one has that-

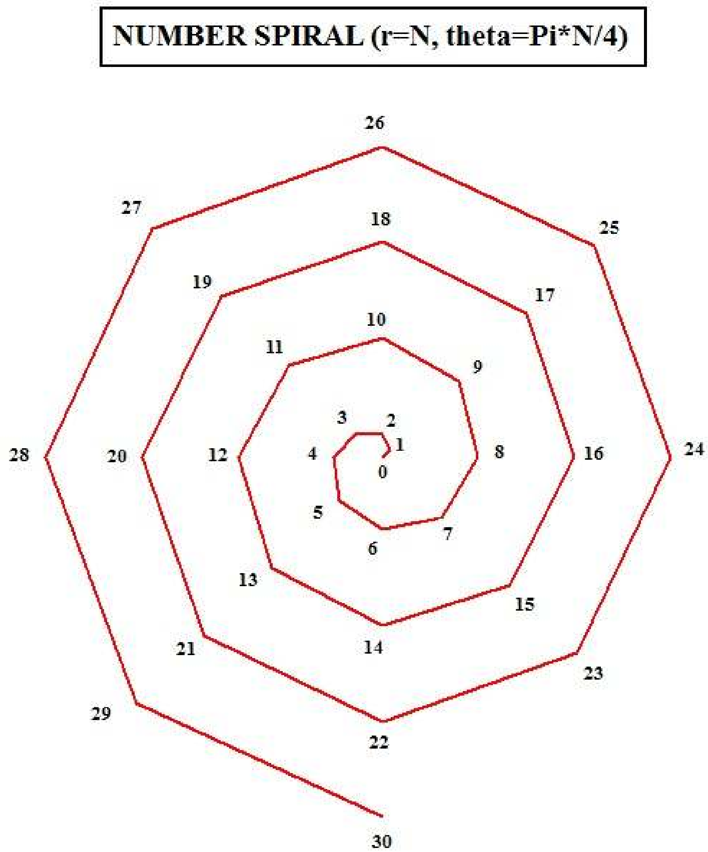
$$L_n = \sqrt{r_n^{\ 2} + r_{n+1}^{\ 2} - 2 r_n r_{n+1} \cos(\theta_n)}$$

Let us work out several examples.

The first of these is the Number Spiral which we discovered several years ago and is defined by the parametric formula-

$$r_N = N \quad and \quad \theta_n = \pi N / 4 \quad N = 0,1,2,3,4,...$$

If we connect these points by straight lines, the following spiral results-



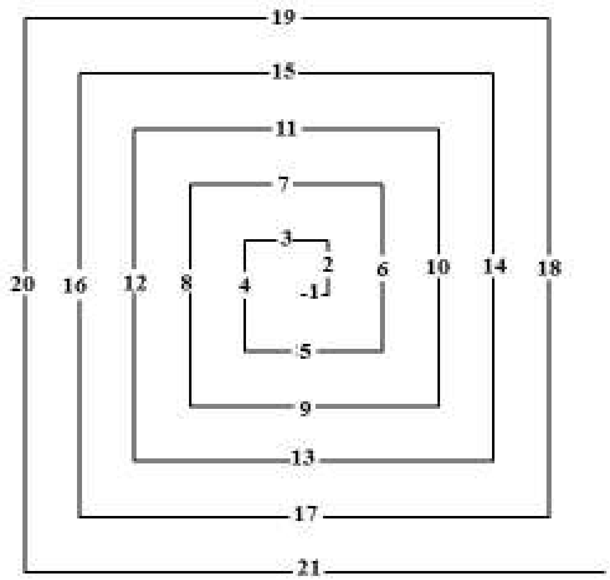
This is a very interesting spiral in which all even numbers lie along the x or y axis. All odd numbers fall along the diagonal lines y=x or y=-x. Since all prime numbers are odd numbers (with the exception of N=2) all prime numbers will also fall along these two diagonal lines.

Another spiral can be constructed by having the segment length and bend angle go as –

$$L_N = N \quad with \quad \varphi_N = \pi / 2$$

This produces-

SQUARE SPIRAL



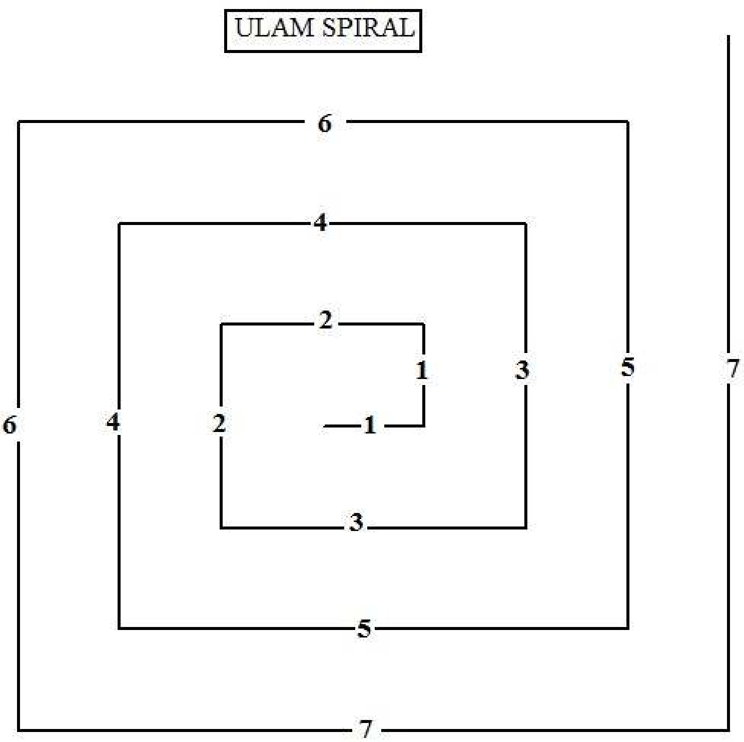
Sidelength L[n]=n

This spiral has side lengths which increase by one unit between segments L_{N+1} and L_N and the curve lengths are nicely separated into even and odd number lengths.

Related to this last spiral is the Ulam Spiral which has $L_1=1$, $L_2=1$, $L_3=2$, $L_4=2$, etc. The bend between segments remains at $\pi/2$ radians counterclockwise. Mathematically we have-

$$L_{2N+1} = L_{2N+2} = N+1 \quad and \quad \varphi_N = \frac{\pi}{2}$$

Its graph looks like this-



Sidelength L[2n+1]=L[2n+2]=n+1

Some people have claimed that this last spiral contains hidden information about prime numbers when looking at the coordinates of its corners, but as we have shown earlier(see our 2008 note http://www2.mae.ufl.edu/~uhk/MORPHING-ULAM.pdf) this is not so since a simple transformation just recasts this spiral into the above Square Spiral. This means any supposed structure found in the Ulam Spiral concerning prime numbers really shows no more than that prime numbers(with the exception of 2) are odd numbers**. It does not say which odd number is prime.**

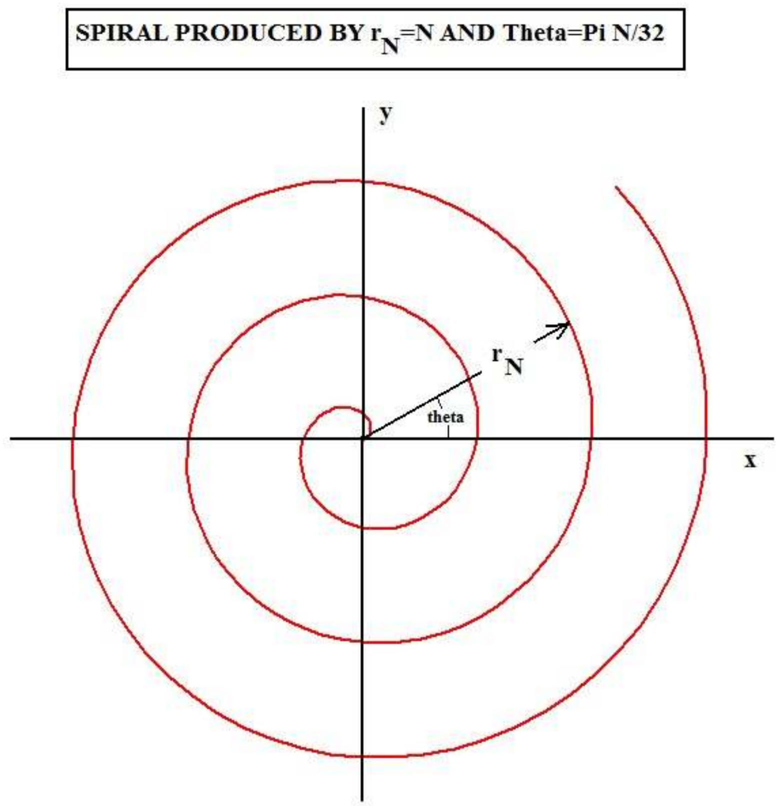
It is possible to come up with an infinite number of other spirals constructed from straight line segments. Among these we have-

$$r_N = N \quad with \quad \varphi_N = \pi N / 32$$

Using the simple MAPLE program

```
with(plots):
listplot([seq([n,(Pi/32)*n],n=0..200)],coords=polar,color=red,thickness=2,axes=none,nu
mpoints=6000,scaling=constrained);
```

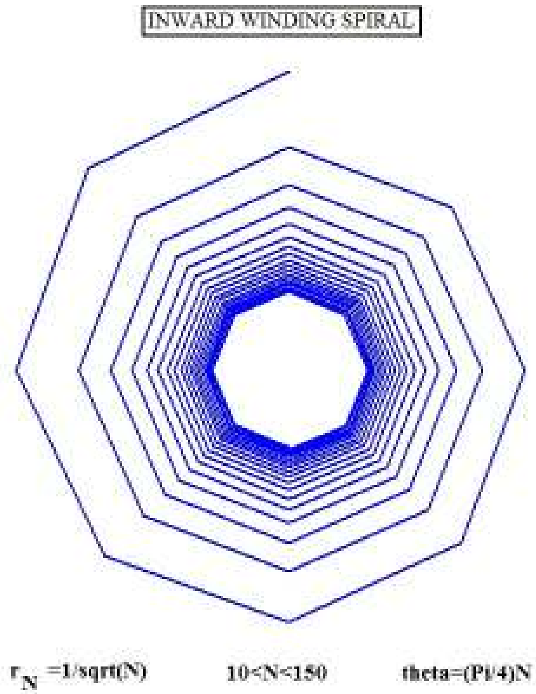
this produces a spiral in which the straight line segments are so short that the curve appears smooth and is essentially a standard Archimedes Spiral $r=(32/\pi)\theta$. We show you here the curve -



Another spiral is the inward winding spiral generated by-

$$r_N = \frac{1}{\sqrt{N}} \quad and \quad \theta_N = \frac{\pi}{4} N$$

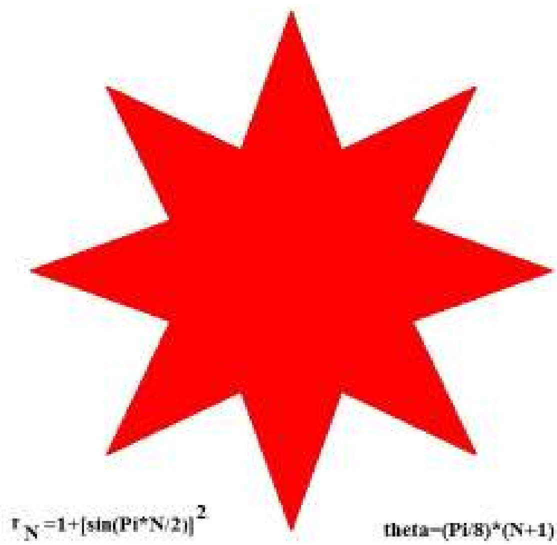
In this case we have $r_{N+1}<r_N$. Over the range $10<N<150$ it produces the pattern-



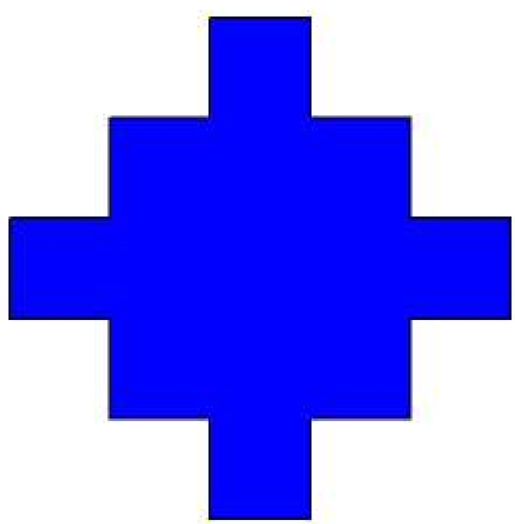
It is also possible to set the radial distance r_N proportional to the familiar Fibonacci Sequence 1, 1, 2, 3, 5, 8, 13,.. while keeping the angle increment constant, This will produce an exponential like spiral.

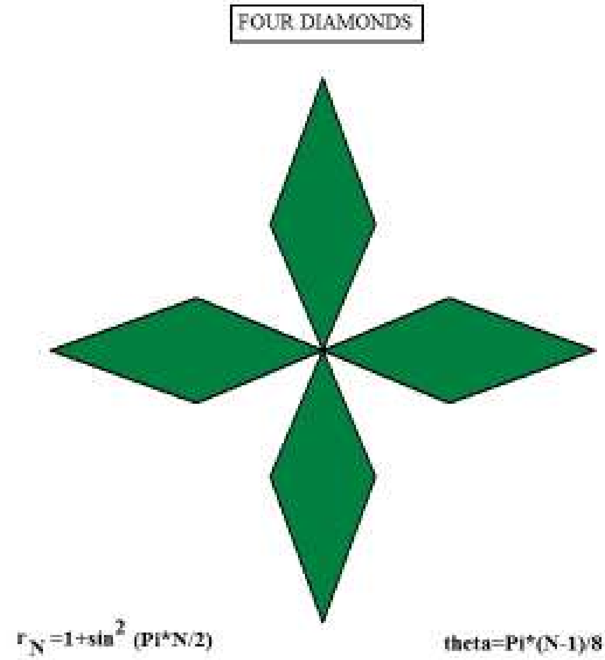
If one is willing to relax the condition that our graphs should represent spirals, then one can obtain a whole class of other figures including things like the Koch Curve and the Dragon encountered in fractal studies. Here are a four examples-

EIGHT POINTED STAR

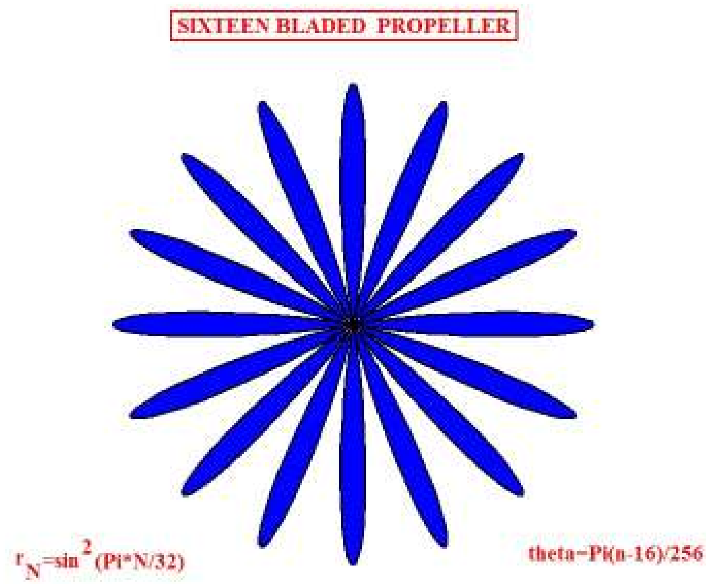


FOUR SQUARES ON ONE LARGE SQUARE





and-



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