TRANSMISSION OF ACCOUSTIC WAVES THROUGH A BARRIER OF FINITE WIDTH

Consider a plane acoustic wave of the form

$$j(x,t)=A_I \exp i(wt-k_1 x)$$

coming from the left and impinging on a barrier extending from x=0 to x=d .
Here j(x,t) is the velocity potential which follows from solving the 1D wave equation and hence the corresponding Helmholtz equation , w is the angular frequency and k_1 is the wave number. The wave propagation speed is $c_1=w/k_1$ and the density of the medium through which the wave propagates is r_1. At the first interface at x=0 there will also be a reflected plane wave of form

$$A_R \exp i(wt+k_1 x)$$

The incoming wave also creates multiple forward and backward waves within the layer $0<x<d$ where the sound speed has the different value c_2 and the density is r_2. We can represent these acoustic waves within the barrier by

$$A_{BR} \exp i(wt-k_2 t)+A_{BL} \exp(i(wt+k_2 t)$$

where one notes that the wave number has changed although the wave frequency stays the same. Finally for x>d there will be just one wave travelling to the right having the form

$$A_T \exp i(wt-k_1 x)$$

Now the interfacial conditions at x=0 and x=d are that the velocity and the pressure must be continous there. Recalling that the spatial derivative of the velocity potential is equivalent to the fluid element velocity and that for small amplitude waves the pressure(from Bernoulli) goes as the negative of the product of fluid density and the time derivaticve of the velocity potential, one obtains four algebraic equations for the five amplitudes A_n. This set becomes soluable for the amplitude ratios $a=A_R/A_I$, $b=A_T/A_I$, $e=A_{BR}/A_I$, and $f=A_{BL}/A_I$. The equations expressed in matrix form reads

$$\begin{bmatrix} 1 & 0 & p & -p \\ 1 & 0 & -q & -q \\ 0 & m & -pn & p/n \\ 0 & m & -qn & -q/n \end{bmatrix} \begin{bmatrix} a \\ b \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

where $p=k_2/k_1=c_1/c_2$, $q=r_2/r_1$, $m=\exp(ik_1d)$, and $n=\exp(ik_2d)$. This equation has a straight forward solution and one finds , for example, that

$$b=\frac{A_T}{A_I}=\frac{4pqn}{m[(p+q)^2-n^2(p-q)^2]}$$

Now, a measure of the energy transmitted through the barrier compared to the energy carried by the incoming wave is known to go as the square of the wave amplitudes and hence the barrier transmission coefficient becomes

$$T=[abs(b)]^2=[abs(\frac{4gn}{m[(1+g)^2-n^2(1-g)^2]})]^2$$

Where $g=r_2c_2/r_1c_1$ is the acoustic impedance ratio. When g is not near unity, then one can generally expect poor transmission of the sound wave. We demonstrate this transmission behaviour for the interesting case of a sound wave in air where r_1=0.001gm/cm3 and c_1=330m/s passing through a d=3mm thick plate glass window where r_2=2gm/cm3 and c_2=5000 m/s. There $g=3\text{x}10^4$. A plot of sound transmission versus the non-dimensional parameter wd/c_2 can be found by clicking on the title to the section containing this pdf file. Note that the glass provides excellent insulation against sound transmission letting only about 1/10 % of the sound energy through at 1000Hz. The lower the sound frequency the more of the sound will be transmitted. This is one of the reasons that it is mainly the low frequency components of teenager car stereos which lead to noise complaints. One also observes that there will be perfect transmission (T=1) at points where wd/c_2 is equal to an integer value of Pi. This is the phenomenon of constructive interference and is one quite familiar to quantum physicists dealing with atomic barrier penetrations.