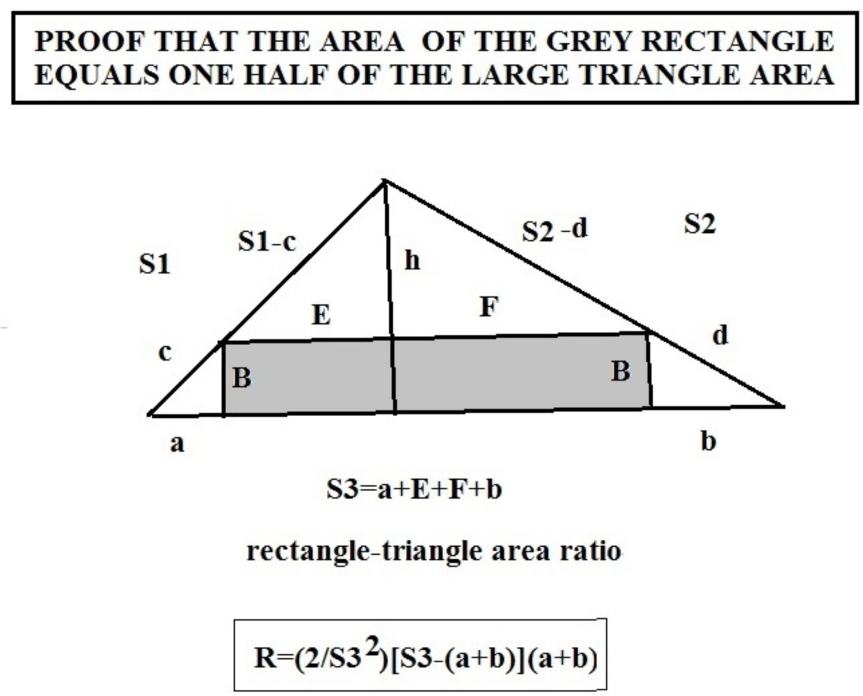
SOLUTION OF A MATH PROBLEM FROM THE SEPTEMBER 30th ISSUE OF THE WALL STREET JOURNAL

A problem appearing in yesterday's Wall Street Journal in their weekend puzzle page deals with finding the maximum area of a rectangle which just fits into a 9-10-17 oblique triangle. We present here a simple solution requiring only the use of similar triangles, plus a little calculus. To begin, we first produce the following schematic of the problem in its most general form-



We have there an oblique triangle with sides S1,S2, and S3. The area of the un-maximized rectangle is shown in grey and has area-

$$A_{rectangle}=B(E+F)$$

By similar triangles we also have-

$$F=(hB)/B \quad \text{and} \quad E=(ha)/B$$

Hence-

$$A_{rectangle}=[S3-(a+b)]B$$

Also the area of the large triangle is-

$A_{triangle}$=(h+B)(a+E)/2+(h+b)(b+F)/2=S3(h+B)/2

But one can also show that (h+B)=[BS3]/(a+b). Hence-

$A_{triangle}$=[$(S3)^2$B]/[2(a+b)]

Thus we find the ratio of the rectangle to triangle area equals-

$$R = \frac{2}{(S3)^2}\left[(a+b)(S3) - (a+b)^2\right]$$

To maximize this result we now take the derivative of R with respect to (a+b) .This produces (a+b)=(S3)/2. From it we arrive at the important conclusion that-

$$R = \frac{2}{(S3)^2}[(S3)^2/2 - (S3)^2/4] = \frac{1}{2}$$

This is a general property for the largest inscribed rectangle which just fits into in any oblique or right triangle.

We can now solve the specific math puzzle appearing in this week's Wall Street Journal. We have-

$A_{rectangle}=A_{triangle}/2$

And the area of any the triangle is, by Heron's Formula, equal to-

$$\sqrt{s(s-9)(s-10)(s-17)} = 36$$

, where the sides have length 9 ,10, and 17 and the semi-perimeter is s=(9+10+17)/2=18 .

So the rectangle of maximum area fitting into this triangle is exactly-

$A_{rectangle}$=36/2=18 square units

In case you are not familiar with the Heron Formula, the same area of 36 for the triangle can be obtained by summing up all the sub-triangles and the rectangle areas shown in the above figure.

It is also possible to solve the puzzle using trigonometry starting with the Law of Cosines results-

$\cos(\alpha)$=15/17 $\cos(\beta)$=3/5 $\cos(\gamma)$=77/85

This leads to the rectangle area-

$$Area=17B-B^2(\cot(\alpha)+\cot(\gamma)]$$

, which when optimized leads to the same result of-

$$A_{rectangle}=18 \text{ square units}$$

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