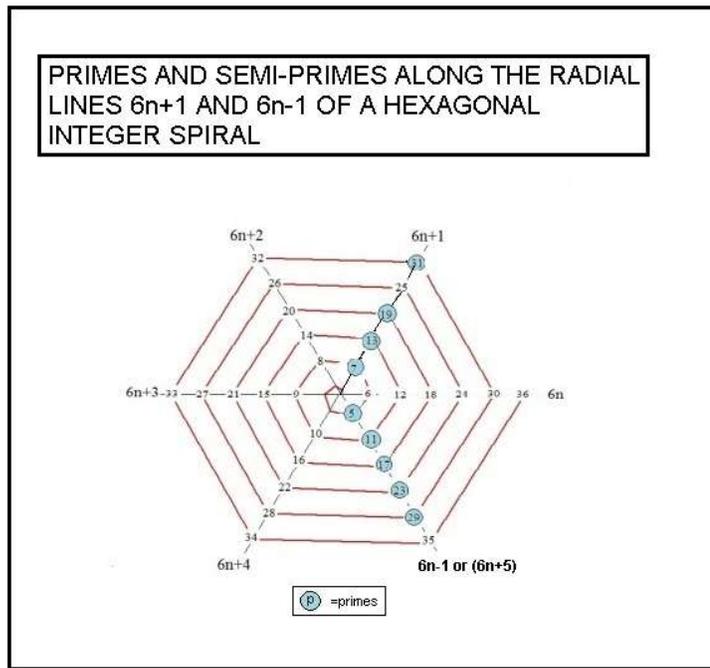


GENERATING TRIPLE PRIMES

We have shown in numerous earlier articles on this TECH-BLOG page that a necessary but not sufficient condition that a number N is prime is that $N=6n\pm 1$, with integer n equal to one or greater. These primes are shown in blue at the crossing points of the radial lines $6n+1$ and $6n-1$ with a hexagonal integer spiral as shown-



This new and far improved version of an Ulam Spiral was first discovered by us about a decade ago. It has taken nearly a decade for mathematicians to recognize the improvement of such a prime representation not present in earlier versions where the primes are much more scattered. You will notice that there are a few gaps, such as 25 and 35, along these radial lines. Such gaps represent the location of Semi-Primes $N=pq$, Tripple-Primes, etc. It is our purpose here to look at some of the properties of Triple Primes which we define as-

$$T=pqr \quad \text{with } p, q \text{ and } r \text{ each a prime five or greater.}$$

To generate these T s we begin with the two prime sequences-

$$6n+1=\{7,13,19,25,31,37,43,49,55,61,67,73,79,85,91,97,103,109,115,121,127,133,139,145,151,157,163,169,175,181,\dots\}$$

$$6n-1=\{5,11,17,23,29,35,41,47,53,59,65,71,77,83,89,95,101,107,113,119,125,131\}$$

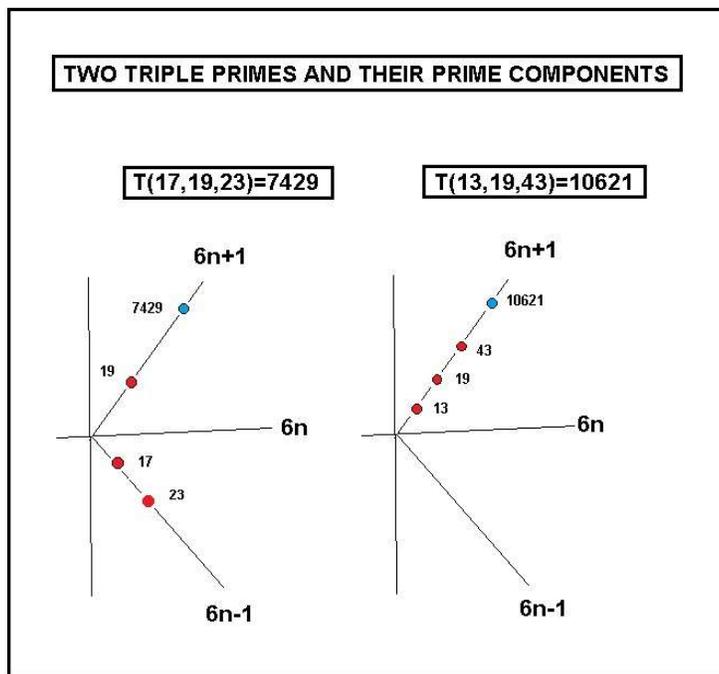
The gaps marked in red are semi-primes, while the Triple-Primes, appearing in blue, are -

$$T(5,5,5)=125 \quad \text{and} \quad T(5,5,7)=175$$

You will note that the density for Triple-Primes is considerably less than that for Semi-Prime. Examples of other Triple Primes are -

$$T(5,7,11)=385, \quad T(7,11,13)=1001, \quad \text{and} \quad T(17,19,23)=7429$$

Observe that the Ts all end in an odd integer. This is expected since the product of three primes is always an odd number if the size restriction for each prime remains at $P \geq 5$. Notice that $T(\text{mod}6)$ is either equal to 1 or 5 meaning it lies along the $6n+1$ or $6n-1$ radial lines, respectively. The geometric pattern associated with two Triple-Primes follow-



Here the Ts are drawn in blue while the three contributing primes are shown in red.

Unlike for Semi-Primes, knowledge of the sigma of T does not allow a direct analytic factoring of T as there are one too many variables. However one can try a brute force approach as follows. Start with a Triple-Prime $T=2093$ and carry out the division by n until a new integer is found. Here we have $2093/7=299$. Next carry out $299/m$. You will find the solution $299/13=23$. Thus your triple prime factors as $T=2093=7 \times 13 \times 23$. Note that here 7 is slightly less than the cube root of T. This fact helps in restricting the range of n to be used for the smallest of the three prime searches in T. It is possible to write this last procedure as-

$$T=2093=7 \times 299=7 \times (13 \times 23)$$

Here is another factored Triple-Prime-

$$T=63181=23 \times 2747=23 \times (41 \times 67)$$

Note that $23 < 39.82$.

An alternative way to factor Triple-Primes is to carry out an ifactor operation on T using an advanced math program such as MAPLE. Applying this approach yields-

$$T= 110615591899837 =56747 \times 62323 \times 31277;$$

in less than a split second. Here –

$$T(\text{mod}6)=1, \quad 56747(\text{mod}6)=5, \quad 62323(\text{mod}6)=1, \quad 31277(\text{mod}6)=5 \quad .$$

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July 7, 2023
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