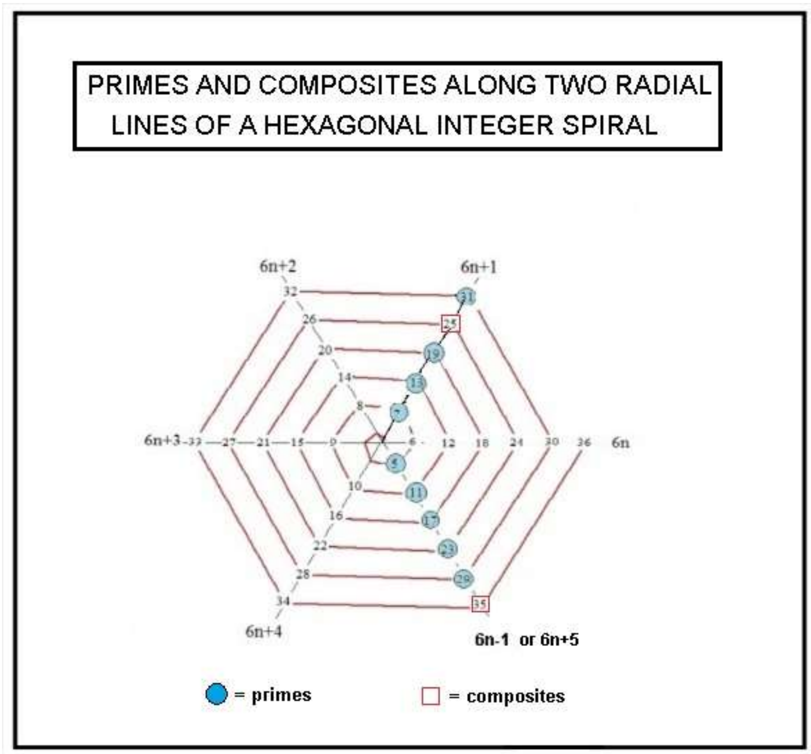
A NECESSARY AND SUFFICIENT CONDITION FOR ALL PRIMES

A little over a decade ago we came up with a new way to plot all positive integers including prime numbers as lying at the intersections of a hexagonal integer spiral and two intersecting radial lines 6n±1 as shown-



In this picture the primes are indicated by blue circles and those in the red boxes as composdites, lying along these same two radial lines,. This result led us to the conclusion that a necessary but not sufficient condition is that all primes five or greatere must have the form 6n±1. That is, N=6n+1 if N mod(6)=1 and N= 6n-1 if N mod(6)=5. As an example , we look at the twenty digit number –

N= 32983103315292673181 for which N mod(6)=5

This means that N= 6(5497183885882112197) -1 .

Since its discovery, we have used this hexagonal integer result in a variety of areas including noticing that twin primes must have a mean value of 6n and that no triple primes can exist if the components differ

by only two units each. Furthemore the distance between any two primes along a radial line differ from each other by a multiples of six. Thus 31-7= 4*6.

We want in this note to strengthen the above prime number criterion by adding something to make it both necessary and sufficient.

We begin our analysis by looking at the following table-

Primes(blue) and Composites(red) along radial lines $6n\pm1$

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
6n+1	7	13	19	25	31	37	43	49	55	61	67	73	79	85	91	97
6n-1	5	11	17	23	29	35	41	47	53	59	65	71	77	83	89	95

Here we give the values of 6n+1 and 6n-1 for the first sixteen integers n. The primes are typed in blue and the composites in red. We next introduce the sigma function from number theory. This point function is defined as –

σ(N)=sum of all divisors including 1 and N

The composites in the table are N=25, 35, 49, 55, 65,77, 85, and 91. The rest are primes. Note now that if N is a prime then it must be true that-

$$\sigma(N) - N=1$$

So we can conclude that we have ***a necessary and sufficient condition for a number N five or greater to be prime is that N equals 6n+1 or 6n-1 plus σ(N)-N=1***.

We are now in a position to test this criterion for several different numbers N.Take first the twenty digit N given above. There the number satisfies 6n-1 so that N mod(6)=5 and my computer yields sigma(N)= 329103315292673182. So we also have sigma(N)-N=1 . This makes p = 32983103315292673181 a prime number.

Next consider the number-

N =6(8345641)+1= 50077347 where N mod(6)=1

This number yields σ(N)-N=1222409>1. Hence N=50077347 is a composite. We never had to show that this composite also equals the exact value –

N=47*367*2903

As a last example of a possible prime, consider-

N=2^32+1=4294967297

This is the famous Fermat number which he thought was a prime but which Euler proved later to be a composite. Let us quickly analyze the number. We have N mod(6)=5 hence N=6(715827883)-1. Also sigma(N)= 4301668356, so that sigma(N)-N=6702059>1.Hence we have that N is a composite. Euler, after months of pre-computer effort, actually showed that –

N=F(5)=2^32+1=641 x 6700417

The next lower Fermat number F(4)=2^16+1=65537 is indeed a prime number since F(4) mod(65537^)=5 so that1 N=6(10923)-1 and sigma(N)=65538 so that 65538-65537=1.

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